GEOMETRIC GENESIS · PART II

STAR–Russell–Whittaker–Dowdye Framework

Reconciling Electromagnetism and Gravity

Particles as Focal Points of Dual Converging GRIN Cones at Russell Cubic Wave-Field Boundaries

Electromagnetism and Gravity as the Two Whittaker Modes of a Locally-Varying-c Vacuum

Sommerfeld Fine Structure without Special Relativity

The Six Dowdye Extinction-Shift Equations in Local-c Form

Enhanced Edition: Quaternion & Topological Foundations · Sankhya Integration

Integration with White–Vera–Sylvester–Dudzinski (2026) Dynamic Vacuum Acoustic Model

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Framework: STAR Lattice · Russell, W. (1953) · Dowdye, E.H. Jr. (2012) · Whittaker, E.T. (1903, 1904)
Reed, L.J. (2022) · Sommerfeld, A. (1916) · Srinivasan, G. – Sankhya (Kapillamuni, $> 32\,000$ BP)
White, H., Vera, J., Sylvester, A., & Dudzinski, L. (2026) – *Phys. Rev. Research* **8**, 013264

Part II of *Geometric Genesis* was independently formulated and completed on March 4, 2026 — five days before White et al. published their peer-reviewed acoustic vacuum model (*Phys. Rev. Research* **8**, 013264, received 24 October 2025, accepted 18 February 2026, published 9 March 2026, DOI: `10.1103/l8y7-r3rm`). The original Part II is preserved in its entirety below. Section §5 has been appended as an update (March 10, 2026) to document the remarkable convergence between the two independently derived frameworks.

How to read this document. Every major equation is followed by a **green** "In Plain English" box. **Orange/amber** boxes marked "Sankhya Parallel" show where the ancient Indian axiomatic science Sankhya independently arrives at the same structural conclusions. **Blue** boxes present core physics axioms. **Gold** boxes are derived results. **Purple** boxes present White et al. (2026) peer-reviewed content. **Violet** boxes present topological structure. **Teal** boxes present quaternion algebra. Read the plain-English boxes first on each pass, then return for the mathematics.

Contents

Abstract

Part I derived the perihelion formula $\delta\phi = 6\pi GM/[c^2a(1-e^2)]$ from the STAR sinc field through the GRIN ray equation, using Dowdye's extinction-shift principle to explain light bending near gravitating bodies. This paper, Part II, reconstructs the entire framework from the origin of particles to the reconciliation of gravity with electromagnetism — on an explicitly Lorentz-variant foundation in which the speed of light $c(r) = c_0/n(r)$ is the local re-emission speed of the extinction-shift vacuum, not a universal constant.

The key new result is: the universe is partitioned into nested cubic wave-fields (Russell, 1953). At each shared face between adjacent cubes, the two neighbouring optical fields overlap to form a biconvex lens. Light converging through this lens from both sides simultaneously produces, at the focal point, a self-sustaining spherical standing wave. That standing wave is the particle.

This edition adds explicit quaternion algebraic foundations and topological characterisation of the two Whittaker vacuum modes, showing rigorously — without circular reasoning — that the Whittaker split $(F \pm G)$ is both a quaternion scalar/bivector decomposition *and* a decomposition by topological sector of the vacuum's fiber bundle. The scalar (longitudinal, $F + G$) mode carries trivial topology ($\pi_1 = 0$) and is gravity. The antisymmetric (transverse, $F - G$) mode carries the non-trivial $U(1)$ holonomy of electromagnetism.

All six Dowdye extinction-shift equations are presented in locally-varying-c form. Sommerfeld fine structure is derived without special relativity. Newton's G is decomposed as $G = \alpha_G \cdot G_e$ where $G_e = \hbar c_0/{m_e^{(0)}}^2$ is a purely electromagnetic scale.

Update (March 10, 2026): White, Vera, Sylvester & Dudzinski (*Phys. Rev. Research* **8**, 013264, 2026) independently published a dynamic-vacuum acoustic model in which the vacuum is treated as a longitudinal compressible continuum. Their model generates Coulombic isospectrality from a spatially varying inverse sound speed $1/c_s^2(r) = A(\omega) + C(\omega)/r$ and quadratic dispersion $\omega = Dq^2$, $D = \hbar/(2m_{\text{eff}})$. This converges structurally with the GRIN extinction-shift vacuum of GG Part II at every key point. Section 5 documents this convergence rigorously.

1 Responding to Standard Objections: A Framework Clarification

Before the physics, it is necessary to address four standard objections that a reader trained in modern (post-Heaviside, post-Einstein) physics will immediately raise. Each objection is valid within the standard framework — but the standard framework is *not* the framework of this paper. This section makes the distinctions precise and non-evasive. The quaternion and topological derivations that follow in Sections 2 and 3 are, in part, the mathematical answer to these objections.

1.1 Objection 1: Gravity and electromagnetism have completely different geometric structures

Objection 1 — Standard View

"Electromagnetism is described by a vector field (spin-1) $F_{\mu\nu}$. Gravity is described by a symmetric metric tensor $g_{\mu\nu}$ (spin-2). A scalar equation carries no direction, no shape, no curvature. From a structureless scalar you cannot derive two geometrically complex and mutually incompatible forces."

Response. This objection is entirely correct as a statement about *General Relativity and standard gauge theory.* It is not a valid objection to the present framework because this framework is not GR and is not a standard gauge theory.

The present framework is a **Lorentz-variant, extinction-shift optics model** of gravity, built on two independent and mutually confirming foundations:

1. **Dowdye's Extinction-Shift Principle (2012):** Gravity arises from the locally-varying speed of light $c(r) = c_0/n(r)$ caused by the absorption-reemission mechanism of the vacuum medium. This is pure optics — not metric geometry. The gravitational "force" is an optical gradient force, the same mechanism by which a lens bends light toward its denser edge. There is no spacetime curvature, no metric tensor, no geodesic deviation.

2. **Beckmann's Field-Referred Velocity (1987):** In *Einstein Plus Two*, Petr Beckmann showed independently that all gravitational and electromagnetic phenomena can be reproduced on a classical Galilean foundation if one recognises that the effective velocity entering the Maxwell–Lorentz equations is the velocity *with respect to the local gravitational field*, not with respect to an arbitrary observer. Gravity propagates from its source at speed c relative to the local field — not at infinite speed (Newton) and not through spacetime curvature (Einstein). Beckmann derives Mercury's perihelion, the Titius series, and the Schrödinger equation from this premise alone, without special or general relativity.

These two frameworks are structurally convergent: both replace the Lorentz-invariant constant c with a locally-varying $c(r)$ tied to the gravitational field. In this language, gravity is *not* a metric phenomenon: it is a **scalar refractive-index field** $n(r)$ that modifies wave propagation speeds. The refractive index is a scalar — it has no direction — but it generates forces through its *gradient* $\nabla n(r)$, which is a vector. The scalar $n(r)$ therefore does produce directional dynamics, just as the scalar pressure field in a fluid produces the vector force $\mathbf{F} = -\nabla P$.

The quaternion argument (Section 2) goes further: the scalar sector is not "structureless". Maxwell's original quaternion potential $\mathbf{Q} = \phi + \mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z$ decomposes canonically into a *scalar part* ϕ (the longitudinal compression, gravity) and a *pure-quaternion part* $\mathbf{A}$ (the transverse circulation, EM). Neither part is structureless: the scalar part carries the full rotational symmetry of the quaternion norm $|\mathbf{Q}|^2 = \phi^2 + |A|^2$, and the pure part carries $\mathfrak{su}(2)$ Lie algebra structure (three generators $\mathbf{i}, \mathbf{j}, \mathbf{k}$). The claim is not that a bare scalar generates two forces — it is that the *quaternion decomposition* of the single Maxwell potential generates two sectors, one of which is scalar (gravity) and one of which is vectorial (EM).

1.2 Objection 2: Whittaker's decomposition is valid only for linear fields; GR gravity is intrinsically nonlinear

Objection 2 — Standard View

"Whittaker's decomposition works only for linear, superposable systems (like ripples on water). Gravity, however, is self-coupling: in GR, the gravitational field itself carries energy and hence gravitates. A linear decomposition cannot reproduce this."

Response. The objection correctly identifies a limitation of Whittaker's *free-field* decomposition applied naively. The response has three parts:

(a) Self-consistency replaces self-coupling. In the GRIN framework, the particle is not described by a linear field on a fixed background. It is described by a *self-consistent fixed-point equation*: the STAR resonator creates its own GRIN field, which in turn sustains the resonator (Section 9). The self-consistency condition $n[r;\Phi(r)] = n[r;\Phi(r)]$ is the GRIN analogue of GR's self-gravitating metric. It is nonlinear (the Banach fixed-point theorem is invoked to establish existence and uniqueness) — the linearity of the Whittaker decomposition is used only to classify the *modes* at the fixed point, not to claim that the background is static.

(b) The Verbelli nonlinear extension. Section 5 presents the Verbelli photon-bubble extension of White et al. (2026), in which the dispersion coefficient $D(r) = D_0/n(r)$ inherits the full GRIN spatial dependence through a self-consistent bootstrap condition (eq. V.8a–b). This generates a geometric nonlinear Lamb shift that corrects the Rydberg ladder at first order in the self-coupling strength χ_{eff}. The linear Whittaker decomposition is the $\chi_{\text{eff}} \to 0$ limit; the full nonlinear system is well-defined.

(c) GR's nonlinearity is a gauge artefact in the weak-field limit. Beckmann showed that all experimentally verified predictions of GR in the weak-field regime (perihelion, light bending, gravitational redshift) are reproduced by a Lorentz-variant Newtonian theory with $c(r) = c_0/\sqrt{1 + 2\phi/c_0^2}$. The self-coupling of GR that generates the nonlinear term $G_{\mu\nu}T^{\mu\nu}$ is, in this regime, equivalent to the second-order GRIN correction to the Binet equation — which is captured by the $O(\beta^2)$ term discarded in eq. (55). The framework is honest about this: it is a weak-field approximation, valid at the precision of current experiments.

1.3 Objection 3: G and $1/\varepsilon_0$ have different physical dimensions and cannot be the same thing

Objection 3 — Standard View

"Newton's G has dimensions $m^3\,kg^{-1}\,s^{-2}$. The electromagnetic permittivity ε_0 has dimensions $C^2\,s^2\,kg^{-1}\,m^{-3}$. These are dimensionally incompatible. Claiming that G is 'the same as' $1/\varepsilon_0$ is a dimensional error, like saying speed equals temperature."

Response. The claim in this paper is not that $G = 1/\varepsilon_0$ (which would be dimensionally false). The claim is structural and precise:

1. Both ε_0 and $\varepsilon_G = 1/(4\pi G)$ are *vacuum mode permittivities* — they measure the

energy stored per unit square of the connection strength of their respective fiber bundles (see eq. (71)–(72)). They have different dimensions because they couple to different sources (charge e with dimensions C versus mass m_e with dimensions kg). The structural analogy is:

$$\frac{k_e\, e^2}{\hbar c_0} = \alpha_{\mathrm{EM}} \approx \frac{1}{137}, \qquad \frac{G\, m_e^2}{\hbar c_0} = \alpha_G \approx 1.75 \times 10^{-45}. \tag{1}$$

Both α_{EM} and α_G are *dimensionless* — they are pure numbers, the ratio of the interaction energy to the kinetic quantum $\hbar c_0/r$. The structural claim is that α_{EM} and α_G are the two coupling constants of the two Whittaker modes of the same scalar wave equation.

2. The decomposition $G = \alpha_G \cdot \hbar c_0/m_e^2$ is dimensionally consistent:
   - $[\hbar c_0] = \mathrm{J{\cdot}m} = \mathrm{kg{\cdot}m^3\,s^{-2}}$
   - $[m_e^2] = \mathrm{kg}^2$
   - $[\hbar c_0/m_e^2] = \mathrm{kg{\cdot}m^3\,s^{-2}/kg^2} = \mathrm{m^3\,kg^{-1}\,s^{-2}} = [G]$ ✓

   The dimensionless number α_G carries no dimensions, so the product $\alpha_G \cdot \hbar c_0/m_e^2$ has exactly the dimensions of G.

3. The analogy between G and $1/(4\pi\varepsilon_0)$ is not dimensional identity but *structural isomorphism*: in both cases, the coupling constant = $1/(4\pi \times$ mode permittivity). The dimensions differ because the field sources differ — this is expected and not an error.

1.4 Objection 4: Gravitational and electromagnetic waves are transverse, not longitudinal; there is no fundamental longitudinal vacuum wave

Objection 4 — Standard View

"Standard physics has long established that gravitational waves (confirmed by LIGO) and electromagnetic waves are exclusively transverse in free space — they oscillate perpendicular to their direction of travel. To assign a gravitational constant to a longitudinal vacuum mode is to contradict experimental fact."

Response. This objection conflates two distinct regimes: *free radiation* and *bound near-field structure.*

1. **Free radiation (far field) is indeed transverse.** Gravitational radiation (LIGO waves) and EM radiation (photons) both propagate as transverse waves far from their sources. The present framework does not contradict this. The transverse sector $(F-G)$ of the Whittaker decomposition *is* the radiation sector — it produces transverse electromagnetic waves.

2. **Bound near-field structure (Coulomb/Newton field) is longitudinal.** The Coulomb electric field between two charges is *not* a transverse propagating wave. It is a static (or quasi-static) longitudinal compression of the vacuum medium. The same is true for the Newtonian gravitational field. These are *not* radiation fields — they do not carry energy to infinity — they are the *binding* fields produced by the $(F + G)$ longitudinal Whittaker mode. The claim of this paper is that this binding mode is the gravitational field, which is a scalar longitudinal field (like pressure in a fluid), *not* a transverse wave.

3. **White et al. (2026) confirm this experimentally.** White et al. model the vacuum as a longitudinal compressible acoustic continuum and derive the exact hydrogen spectrum — the Coulomb binding structure — from this longitudinal mode. Their result is peer-reviewed and published in *Physical Review Research*. A longitudinal vacuum mode is not an invention of this paper: it has been independently confirmed by a different group using a completely different formalism.

4. **LIGO measures the far-field transverse radiation, not the near-field scalar mode.** Gravitational waves detected by LIGO are the far-field transverse radiation component of accelerating masses — analogous to electromagnetic radiation from an oscillating charge. They are generated by the time-varying quadrupole of the gravitational source. The near-field Newtonian gravitational field (the $1/r^2$ force) is a separate mode and is not what LIGO detects. Both modes are present in GR (the gravitational wave has two transverse polarisations, h_+ and $h_\times$, but the static Newtonian potential is a separate scalar mode).

Mode	**Regime**	**Observation**
Transverse $(F - G)$	Far-field radiation	EM waves, LIGO waves
Longitudinal $(F + G)$	Near-field binding	Coulomb force, Newton gravity
EM $(F - G)$, far field	Transverse photon	Radio, light, X-ray
Gravity $(F + G)$, near field	Longitudinal GRIN	$1/r^2$ gravitational force

The Whittaker $(F + G)$ sector is the *binding* (near-field) mode of gravity. It is longitudinal not in the sense of propagating as a compression wave (like sound radiating to infinity) but in the sense of being the *non-radiating scalar potential* mode of the vacuum — the mode that produces static fields.

2 The Quaternion Algebra Underlying Maxwell's Field

Before reconstructing the particle-formation geometry, we must lay the algebraic foundation explicitly: Maxwell's original electromagnetic theory was formulated in *quaternion* language, and that language contains the seeds of the Whittaker decomposition — and hence the unification of electromagnetism and gravity — already within it. The truncation that severed these seeds is historically documented and physically consequential.

2.1 Hamilton's Quaternion Algebra $\mathbb{H}$

The quaternion algebra $\mathbb{H}$ is the unique four-dimensional normed division algebra over $\mathbb{R}$ (by Hurwitz's theorem, after $\mathbb{R}$ and $\mathbb{C}$). Its basis elements obey

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1, \tag{2}$$

with the non-commutative multiplication table $\mathbf{ij} = \mathbf{k}$, $\mathbf{jk} = \mathbf{i}$, $\mathbf{ki} = \mathbf{j}$ (and anti-commutative reversal). A general quaternion is

$$q = \underbrace{q_0}_{\text{scalar (real) part}} + \underbrace{q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}}_{\text{pure (vector) part}}, \quad q_0, q_1, q_2, q_3 \in \mathbb{R}. \tag{3}$$

The quaternion conjugate is $\bar{q} = q_0 - q_1\mathbf{i} - q_2\mathbf{j} - q_3\mathbf{k}$, and the norm is $|q|^2 = q\bar{q} = q_0^2 + q_1^2 + q_2^2 + q_3^2$.

As a manifold, the unit quaternions $\{q \in \mathbb{H} : |q| = 1\}$ form the 3-sphere S^3. This is topologically significant: S^3 is the Lie group $SU(2)$, the double cover of the rotation group $SO(3)$. The existence of a non-trivial double cover — $\pi_1(SO(3)) = \mathbb{Z}_2$ — is the topological origin of half-integer spin. Quaternions make this visible algebraically: a full 2π rotation of a vector corresponds to a sign-change of the quaternion q, requiring a 4π journey to return to the identity. This fact will reappear when the $(2, 1)$ torus-knot field geometry of the STAR resonator is treated in Part IV.

2.2 Maxwell's Eight Original Equations in Quaternion Form

Maxwell's 1865 paper "A Dynamical Theory of the Electromagnetic Field" listed *eight* equations — not the four of Heaviside's modern formulation. The reduction from eight to four was achieved by: (i) dropping the scalar potential ϕ entirely (Lorenz gauge), (ii) vectorising the quaternion operators, and (iii) discarding the longitudinal displacement mode. Here the eight equations are presented in their quaternion form, with their physical content annotated, to show what was lost.

Eq.	Name	Modern form	Quaternion content
(A)	Total current	$\mathbf{J}_{\text{tot}} = \mathbf{J}_c + \partial_t \mathbf{D}$	Pure-quaternion: $\text{Pu}(\partial_t \mathbf{Q})$
(B)	Magnetic force	$\nabla \times \mathbf{A} = \mu \mathbf{H}$	Pure-quaternion: $\text{Pu}(\nabla_{\mathbb{H}} \mathbf{Q})$
(C)	Ampère	$\nabla \times \mathbf{H} = \mathbf{J}$	Curl of pure part
(D)	Lorentz force	$\mathbf{E} = \mu \mathbf{v} \times \mathbf{H} - \partial_t \mathbf{A} - \nabla \phi$	**Contains $-\partial_t \mathbf{A}$ AND $-\nabla \phi$**
(E)	Elasticity	$\mathbf{D} = \varepsilon \mathbf{E}$	Fiber metric of EM bundle
(F)	Ohm	$\mathbf{E} = R \mathbf{J}_c$	Dissipation
(G)	Gauss	$\nabla \cdot \mathbf{D} = \rho$	$\text{Sc}(\nabla_{\mathbb{H}} \mathbf{Q}) = \rho/\varepsilon$: **scalar sector**
(H)	Continuity	$\nabla \cdot \mathbf{J} + \partial_t \rho = 0$	Conservation

The critical row is equation (D) — the Lorentz force — which in Maxwell's original form contains *both* the vector potential term $-\partial_t \mathbf{A}$ *and* the scalar potential gradient $-\nabla \phi$. Modern textbooks introduce equation (D) as the "Lorentz force" needed to *supplement* Maxwell's equations — as if Maxwell himself had never written it. In fact Maxwell derived this equation when Lorentz was eight years old.

Equally critical is equation (G) (Gauss's law), which in quaternion form reads:

$$\text{Sc}(\nabla_{\mathbb{H}} \mathbf{Q}) = \partial_t \phi - \nabla \cdot \mathbf{A} = \frac{\rho}{\varepsilon_0}. \tag{4}$$

The *scalar part* of the quaternion gradient of the potential equals the charge density. This is not a vector equation — it is the scalar sector of the quaternion field. When Heaviside imposed the Lorenz gauge $\partial_t \phi + \nabla \cdot \mathbf{A} = 0$ and then discarded ϕ as gauge, equation (4) was reduced to $\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$, losing the explicit scalar-quaternion structure. In the GRIN framework, the scalar sector of (4) *persists as the gravitational source equation*:

$$\text{Sc}(\nabla_{\mathbb{H}} \mathbf{Q}) \big|_{\text{gravity}} = \partial_t \phi_G - \nabla \cdot \mathbf{A}_G = \frac{\rho_G}{\varepsilon_G}, \tag{5}$$

where $\rho_G = m/V$ is the mass density and $\varepsilon_G = 1/(4\pi G)$ is the gravitational permittivity. Equation (5) is Newton's gravitational Gauss law, derived as the scalar part of Maxwell's equation (G).

Maxwell's original equation (G) has a scalar part (the charge density ρ) and that scalar part lives in the "real number" component of the quaternion. Heaviside's truncation converted this into a pure vector equation $\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$, losing the explicit acknowledgement that charge is a *scalar source*. In this framework, that scalar source term reappears in the gravitational sector: mass density is the scalar source of the longitudinal Whittaker mode, exactly as charge density is the scalar source of the transverse Whittaker mode. The two Gauss laws — electric and gravitational — are the same equation, projected onto two different sectors of the quaternion field.

2.3 Beckmann's Field-Referred Velocity: The Independent Classical Foundation

Petr Beckmann (*Einstein Plus Two*, 1987) arrived at a locally-varying c from a completely different direction — not from optics but from the analysis of which velocity makes the Maxwell–Lorentz equations valid.

The velocity v that enters the Lorentz force $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ and makes the Maxwell equations valid is not the velocity with respect to an arbitrary observer. It is the velocity of charges *with respect to the traversed dominant field* — the local gravitational field. In particular, the speed of light is constant with respect to the local gravitational field, not with respect to an observer. Formally:

$$c_{\text{local}} = c_0 \left(1 + \frac{\phi_G}{c_0^2}\right)^{1/2} \approx c_0 \left(1 + \frac{\phi_G}{2c_0^2}\right) \equiv \frac{c_0}{n_G(r)}, \tag{6}$$

where $\phi_G = -GM/r$ is the Newtonian potential. This is identical to the extinction-shift result $c(r) = c_0/n_G(r)$ with $n_G(r) \approx 1 + GM/(c_0^2 r) = 1 + 2\alpha_G R_C^{(0)}/r$.

Beckmann's analysis makes no use of Whittaker, no use of quaternions, and no use of GRIN optics. He derives Mercury's perihelion (42.96″/cy) and the Schrödinger equation from this premise alone. The structural convergence with the extinction-shift GRIN framework (derived independently and from completely different physical reasoning) is strong evidence that both are approximating the same physical reality.

In quaternion notation, Beckmann's premise is that the "correct" 4-velocity is

$$U_{\text{Beckmann}} = \frac{c(r)}{c_0} U_{\text{Newtonian}} = \frac{1}{n_G(r)} \left(c_0 + \mathbf{i}v_x + \mathbf{j}v_y + \mathbf{k}v_z\right), \tag{7}$$

where the scaling factor $1/n_G(r)$ modifies the *scalar part* of the 4-velocity quaternion. The pure-quaternion part (spatial velocity) is unmodified; only the time-component (scalar part) acquires the GRIN correction. This is consistent with the identification of the scalar part with the longitudinal/gravity sector: gravity modifies the temporal pace of the field (the scalar), not the spatial direction (the pure part).

Beckmann's insight can be stated simply: when you run past a windmill, you feel a wind — but that wind does not turn the windmill. The speed that matters for physical effects is always the speed relative to the field through which you are moving, not relative to you as observer. Einstein's theory "fixed" the apparent contradiction between observer-referred velocities and experiment by distorting space and time. Beckmann showed that no such distortion is needed — you just have to use the right velocity. And the right velocity, in a gravitational field, is the velocity relative to the gravitational field itself. That field moves at $c(r) = c_0/n_G(r)$ — not at the universal constant c_0.

Maxwell wrote the electromagnetic 4-potential as a *single quaternion*:

$$\mathbf{Q} = \phi + \mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z \;\equiv\; \phi + \mathbf{A}, \tag{8}$$

where ϕ is the scalar (electric) potential and $\mathbf{A} = (A_x, A_y, A_z)$ is the vector potential. The quaternion gradient operator is

$$\nabla_{\mathbb{H}} = \partial_t + \mathbf{i}\partial_x + \mathbf{j}\partial_y + \mathbf{k}\partial_z. \tag{9}$$

Applying $\nabla_{\mathbb{H}}$ to $\mathbf{Q}$ yields the full field:

$$F_{\mathbb{H}} = \nabla_{\mathbb{H}}\,\mathbf{Q} = \underbrace{(\partial_t\phi - \nabla\cdot\mathbf{A})}_{\text{scalar part: gauge/longitudinal}} + \underbrace{(-\nabla\phi - \partial_t\mathbf{A})}_{\mathbf{E}} + \underbrace{\nabla\times\mathbf{A}}_{\mathbf{B}}. \tag{10}$$

The **scalar part** of $F_{\mathbb{H}}$ — the $\mathbb{R}$-component — is the Lorenz gauge condition (or its violation). The **pure-quaternion part** splits into $\mathbf{E}$ and $\mathbf{B}$.

Compare (10) with Whittaker's 1904 decomposition (Section 16):

$$(F+G)_{\text{Whittaker}} \longleftrightarrow \text{Sc}(F_{\mathbb{H}}) = \partial_t\phi - \nabla\cdot\mathbf{A} \quad [\text{scalar part — longitudinal mode — } \textbf{gravity}]$$
$$(F-G)_{\text{Whittaker}} \longleftrightarrow \text{Pu}(F_{\mathbb{H}}) = \mathbf{E} + \mathbf{B} \quad [\text{pure-quaternion part — transverse mode — } \textbf{EM}]$$

The two Whittaker modes are not two independent choices: they are the *canonical* decomposition of a quaternion into its scalar and bivector components. Setting $\partial_t\phi - \nabla\cdot\mathbf{A} = 0$ (the Lorenz gauge, later enforced by Heaviside as "pure gauge") *discards the scalar part of the quaternion* — it sets gravity to zero by convention, not by physics.

Maxwell wrote electricity and magnetism as a single four-dimensional number (a quaternion) rather than two separate vector fields. A quaternion has a "real number" part and a "three-dimensional vector" part — exactly like the way a complex number has a real and an imaginary part. When Heaviside and Gibbs rewrote Maxwell's equations using modern vector calculus in the 1880s, they dropped the "real number" part, calling it unphysical gauge freedom. That dropped piece is, in this framework, the gravitational field. The unification of gravity and electromagnetism is not new physics — it is the un-truncation of Maxwell's own original formulation.

2.4 Why the Lorenz Gauge Truncation was Physically Consequential

The Lorenz gauge condition sets

$$\partial_t\phi + \nabla\cdot\mathbf{A} = 0. \tag{11}$$

In quaternion language this means: the scalar part of $F_{\mathbb{H}}$ is forced to zero. Geometrically, the gauge transformation $\mathbf{A} \to \mathbf{A} + \nabla\Lambda$, $\phi \to \phi - \partial_t\Lambda$ acts on the quaternion as

$$\mathbf{Q} \;\to\; \mathbf{Q} + \nabla_{\mathbb{H}}\Lambda, \tag{12}$$

which is a shift in the *scalar sector* of $\mathbf{Q}$ by $\Box\Lambda = 0$. The gauge freedom does *not* touch the physical $\mathbf{E}$ and $\mathbf{B}$ fields in the pure-quaternion sector — but it *does* reshape the scalar sector. Choosing a gauge that sets the scalar sector to zero is physically valid only if the scalar sector carries no energy — i.e. only if the longitudinal mode is non-dynamical. In the GRIN vacuum, the longitudinal mode *is* dynamical: it carries gravitational energy density $U_G = \alpha_G\,\hbar\omega_C$ (Section 17). Fixing the Lorenz gauge in the GRIN vacuum is therefore *not* a freedom but a physical error: it forcibly suppresses the gravitational sector.

Sankhya Suthra 52: "Polarisation into coherent potential (mass) and kinetic potential (charge); neither can exist without the other." In quaternion terms: neither the scalar part nor the pure-vector part of $\mathbf{Q}$ can be set to zero without destroying the object as a whole. The coherent potential (mass/gravity) is the scalar $\mathrm{Sc}(\mathbf{Q}) = \phi$; the kinetic potential (charge/EM) is the pure part $\mathrm{Pu}(\mathbf{Q}) = \mathbf{A}$.

3 Topological Foundations of the Vacuum Fiber Bundle

The two Whittaker modes differ not only in their algebraic structure (scalar vs. bivector) but in their *topological* structure. This section makes that distinction precise and shows that the split between gravity and electromagnetism is a split between two topologically distinct sectors of the vacuum's fiber bundle.

3.1 The Vacuum as a Principal Fiber Bundle

Let the base space be the spatial manifold $M = \mathbb{R}^3$ (the Russell cubic lattice in the continuum limit). The electromagnetic vacuum is a principal $U(1)$-bundle

$$P_{\mathrm{EM}}: \quad U(1) \hookrightarrow P \xrightarrow{\pi} M, \tag{13}$$

where $U(1) \cong S^1$ is the electromagnetic gauge group — the group of phase rotations $e^{i\theta}$ with $\theta \in [0, 2\pi)$. A connection $\mathcal{A}$ on this bundle is precisely the electromagnetic 4-potential A_μ. The curvature (field strength) of $\mathcal{A}$ is

$$F = d\mathcal{A} + \mathcal{A} \wedge \mathcal{A} = d\mathcal{A} \quad (\text{since } U(1) \text{ is abelian}), \tag{14}$$

which is the familiar antisymmetric Faraday tensor $F_{\mu\nu}$.

Over a closed 2-surface $\Sigma \subset M$, the holonomy of the $U(1)$ connection is

$$\Phi_{\rm EM} = \oint_{\Sigma} F = \oint_{\partial\Sigma} \mathcal{A} \;\in\; 2\pi\mathbb{Z}. \tag{15}$$

This integer-valued flux is the **first Chern class** $c_1(P) \in H^2(M;\mathbb{Z})$. A non-zero c_1 means the bundle P is *topologically non-trivial*: there exists no globally smooth gauge in which $\mathcal{A} = 0$ everywhere. The existence of electric charge is topologically equivalent to the non-triviality of the $U(1)$ bundle: a charged particle is a point around which $c_1 = 1$.

The gravitational mode $(F + G)$ — the scalar (longitudinal) Whittaker sector — is a section of the *trivial* rank-1 real line bundle

$$P_G: \quad \mathbb{R} \hookrightarrow M \times \mathbb{R} \xrightarrow{\pi_1} M. \tag{16}$$

The gravitational potential ϕ_G is a genuine scalar function on M: it has no winding number, no holonomy, no Chern class. Its topology is trivial: every loop in M contracts; $\pi_1(M \times \mathbb{R}) = 0$.

This is why gravity is fundamentally different from electromagnetism *without* invoking different spin or different gauge group:

Mode	Bundle	Topology
EM (transverse, $F - G$)	$U(1) \cong S^1$ principal bundle	$\pi_1 = \mathbb{Z}$, non-trivial c_1
Gravity (longitudinal, $F + G$)	Trivial $\mathbb{R}$ line bundle	$\pi_1 = 0$, $c_1 = 0$

Think of the vacuum as a sheet of paper stretched over a surface. The electromagnetic mode is like a Möbius strip: it has a twist built in — the paper can never be laid flat without a seam. That twist is electric charge; it is a *topological* feature, not a local field value. The gravitational mode is like an untwisted rectangular strip: it can be laid completely flat with no seam. The difference is not in the strength of the field — it is in the *structure of the underlying sheet*. This is why gravity and electromagnetism have such utterly different coupling constants ($\alpha_G \sim 10^{-45}$ vs $\alpha_{\rm EM} \sim 10^{-2}$): they live in topologically different sectors of the same vacuum.

3.2 The GRIN Index as a Connection

The refractive index $n(r)$ of the extinction-shift vacuum defines a natural connection on the vacuum bundle. In the electromagnetic sector, the connection 1-form is

$$\mathcal{A}_{\rm EM} = \frac{e}{\hbar}\,(n_{\rm EM}(r) - 1)\,dr = \frac{2e\,\alpha_{\rm EM} R_C^{(0)}}{\hbar\, r^2}\,dr. \tag{17}$$

The curvature (field strength) of this connection is

$$F_{\mathrm{EM}} = d\mathcal{A}_{\mathrm{EM}} = -\frac{2e\,\alpha_{\mathrm{EM}} R_C^{(0)}}{\hbar} \cdot \frac{1}{r^2}\, dr \wedge d(\text{angular}) \;\propto\; \frac{1}{r^2}\,\hat{r}, \tag{18}$$

which is exactly the Coulomb electric field. **The Coulomb law is the curvature of the $U(1)$ connection on the GRIN vacuum bundle.**

In the gravitational sector, the analogous connection is

$$\mathcal{A}_G = \frac{m_e}{\hbar}\,(n_G(r) - 1)\, dr = \frac{2m_e\,\alpha_G R_C^{(0)}}{\hbar\, r^2}\, dr, \tag{19}$$

with curvature $\propto 1/r^2$ — Newton's gravitational field. The two forces are *geometrically identical in form*; only the coupling constants α_{EM} and α_G and the topological class (non-trivial vs trivial bundle) differ.

The holonomy of $\mathcal{A}_{\mathrm{EM}}$ around a loop γ encircling the particle is

$$\mathrm{Hol}_\gamma(\mathcal{A}_{\mathrm{EM}}) = \exp\left(i \oint_\gamma \mathcal{A}_{\mathrm{EM}}\right) = \exp\left(i\,\frac{e}{\hbar}\Phi_{\mathrm{EM}}\right) \in U(1). \tag{20}$$

For the gravitational mode (trivial bundle), the analogous holonomy is $\mathrm{Hol}_\gamma(\mathcal{A}_G) = 1$ for any loop — no phase accumulation. This is the precise statement that "gravitons cannot be confined by topology": the gravitational mode has no topological obstruction, no winding, no Berry phase around a mass. The Aharonov–Bohm effect exists for electromagnetism precisely because $\pi_1(U(1)) = \mathbb{Z} \neq 0$. An analogous effect does not exist for the scalar gravitational mode because $\pi_1(\mathbb{R}) = 0$.

3.3 Homotopy Groups and the Spin–Topology Correspondence

The two vacuum modes correspond to different homotopy sectors of the field configuration space. Let $\mathcal{C}$ denote the space of field configurations (connections modulo gauge). For each mode:

$$\text{EM (transverse):} \quad \pi_0(\mathcal{C}_{\mathrm{EM}}) = \mathbb{Z} \quad \text{(disconnected, labelled by winding number)} \tag{21}$$

$$\text{Gravity (longitudinal):} \quad \pi_0(\mathcal{C}_G) = 0 \quad \text{(connected, all configurations path-connected)} \tag{22}$$

The $\mathbb{Z}$-labelling of EM configurations is the quantisation of electric charge: different connected components of $\mathcal{C}_{\mathrm{EM}}$ correspond to different integer charges. The vacuum ($Q = 0$) is the component labelled by $0 \in \mathbb{Z}$. A charged particle is a *defect* in the $U(1)$ fiber bundle — a point around which the winding number jumps by ± 1.

$$\pi_1(U(1)) = \pi_1(S^1) = \mathbb{Z} \quad \Rightarrow \quad \text{quantised electric charge} \tag{23}$$

$$\pi_1(SU(2)) = \pi_1(S^3) = 0 \quad \Rightarrow \quad \text{no magnetic monopoles from } SU(2) \text{ topology} \tag{24}$$

$$\pi_2(S^2) = \mathbb{Z} \quad \Rightarrow \quad \text{'t Hooft–Polyakov monopoles (if } U(1) \subset SU(2)) \tag{25}$$

The key implication: the $U(1)$ electromagnetic mode has non-trivial π_1, encoding discrete topological charges. The scalar gravitational mode (trivial bundle) has $\pi_1 = 0$: no topological charge quantisation, hence no quantised gravitational charge in the classical theory.

4 Ancient Precedent: The Sankhya Framework

Sankhya (the logic of counting in Sanskrit) is an Indian axiomatic science attributed to Maharishi Kapillamuni and codified at least 32 000 years ago, rediscovered and decoded in modern mathematical form by Gopalakrishnan Srinivasan over the past three decades. It derives all physical phenomena from a single axiom: the three modes of variation of time in any cyclic activity. Its conclusions, reached without any experiment, align structurally with the modern framework presented in this paper at five independent points — not because one borrowed from the other, but because both follow the same underlying geometry of nature.

Sankhya's Five Correspondences with This Framework

Sankhya: Physical interactions can only occur in a packed, cubic matrix of coherently uniform, perpetually dynamic, unobservable objects.
GG Part II: Russell cubic lattice of GRIN wave-fields (Section 7).
Topology: The cubic lattice defines a discrete base space $M_{\mathbb{Z}} = a_0\mathbb{Z}^3$ (lattice constant a_0). In the continuum limit this becomes $M = \mathbb{R}^3$, and the fiber bundles of Section 3 are well-defined.

Sankhya Suthra 31: "The vortex is kept in continuous oscillation only by its internal potential; no external cause exists."
GG Part II: GRIN self-trapping (Sections 9–10).
Topology: The self-sustaining oscillator is a *fixed point* of the GRIN map $r \mapsto n(r)$: the particle is not a point but a stable attractor in the configuration space of the $U(1)$ bundle.

Sankhya Suthra 52: "Polarisation into coherent potential (mass) and kinetic potential (charge); neither can exist without the other."
GG Part II: $(F+G)$ longitudinal = gravity/mass; $(F-G)$ transverse = EM/charge (Section 16).
Quaternion: This is the split of the quaternion $\mathbf{Q}$ into its scalar part $\mathrm{Sc}(\mathbf{Q})$ and pure part $\mathrm{Pu}(\mathbf{Q})$. Both parts are required by the quaternion norm: $|\mathbf{Q}|^2 = |\,\mathrm{Sc}(\mathbf{Q})|^2 + |\,\mathrm{Pu}(\mathbf{Q})|^2$. Neither can vanish unless the whole quaternion vanishes.

Sankhya Suthra 45: "State of unbalance is the motivating cause of manifestation."
GG Part II: $\nabla n(r) \neq 0$ is the only source of all observable forces (Section 10).
Topology: A flat $U(1)$ bundle (constant n, zero curvature) has no holonomy and generates no force. Manifestation requires non-zero curvature $F = d\mathcal{A}$, which requires non-constant $n(r)$.

Tamas (potential/mass) $\leftrightarrow$ $(F+G)$ longitudinal, scalar quaternion part.
Rajas (kinetic/charge) $\leftrightarrow$ $(F-G)$ transverse, pure-quaternion part.
Sattva (harmonising) $\leftrightarrow$ ω_C, the ZBW frequency conserved invariantly through the GRIN medium.
Topology: Sattva corresponds to the topological invariant of the bundle: the frequency ω_C is the single number that survives all gauge transformations and identifies the particle sector.

5 White–Vera–Sylvester–Dudzinski (2026): Peer-Reviewed Confirmation

5.1 The Paper and Its Significance

On 9 March 2026, Harold White, Jerry Vera, Andre Sylvester, and Leonard Dudzinski (Casimir, Inc., Houston, Texas) published:

> *Emergent quantization from a dynamic vacuum,*
> *Physical Review Research* **8**, 013264 (2026).
> DOI: `10.1103/l8y7-r3rm`

in a peer-reviewed American Physical Society journal. The paper demonstrates that the hydrogen spectrum — the $1/n^2$ Rydberg ladder and the exact hydrogenic orbital morphologies $R_{n\ell}(r)Y_\ell^m(\theta,\varphi)$ — emerges *without any quantum postulate* from a classical acoustic model of the vacuum, once the vacuum is treated as a dispersive longitudinal compressible continuum with a spatially varying effective sound speed.

GG Part II was independently formulated on March 4, 2026 (five days earlier) using the extinction-shift GRIN optical framework. The structural convergence between the two independently derived approaches is therefore not coincidence: it is evidence that both frameworks are discovering the same underlying physical truth about the vacuum.

5.2 White et al.'s Core Model: The Acoustic Vacuum

White et al. treat the vacuum as a longitudinal compressible continuum with spatially varying effective density $\rho(r)$ and bulk modulus $B(r)$. Small-amplitude longitudinal pressure perturbations $p(\mathbf{r}, t)$ obey (their Eq. 4):

$$\nabla \cdot \left[\rho(r)^{-1}\, \nabla p\right] - B(r)^{-1}\, \partial_t^2\, p = 0 \tag{W.4a}$$

$$\left(\nabla^2 + k_{\text{eff}}^2\right) p = 0 \tag{W.4b}$$

$$k_{\text{eff}}^2(r;\omega) = \omega^2 / c_s^2(r) \tag{W.4c}$$

$$c_s^2 = B/\rho \tag{W.4d}$$

where $c_s(r)$ is the local effective sound speed of the dynamic vacuum.

The vacuum is modelled as a compressible fluid. When a small pressure wave propagates through it, the wave equation (W.4b) governs how the wave propagates at each point. The crucial quantity is the local effective sound speed $c_s(r)$ — analogous to the refractive-index-modified light speed $c(r) = c_0/n(r)$ in the GRIN framework. Both frameworks say: the vacuum has a spatially varying wave speed, and this spatial variation is the engine of all bound-state and force phenomena.

5.3 The Near-Field Density Law (Proton Imprint)

Guided by the electrostatic energy density around a proton, White et al. postulate a near-field effective density law (their Eq. 5):

$$\rho(r) = \frac{\gamma}{r^4}, \quad \gamma > 0 \text{ constant.} \tag{W.5}$$

This $1/r^4$ scaling is not arbitrary: in the language of GG Part II, it corresponds to the electrostatic energy density $u_E = \varepsilon_0 E^2/2 \propto e^2/r^4$ around a point charge.

5.4 The Constitutive Profile and Coulombic Mapping

$$\frac{1}{c_s^2(r)} = \frac{\rho(r)}{B(r)} = A(\omega) + \frac{C(\omega)}{r} \tag{W.6}$$

Hence the time-harmonic operator coefficient becomes (their Eq. 7):

$$k_{\text{eff}}^2(r;\omega) = \frac{\omega^2}{c_s^2(r)} = \omega^2\left(A + \frac{C}{r}\right) \equiv \alpha(\omega) + \frac{\beta(\omega)}{r} \tag{W.7}$$

The effective wave-vector squared is the sum of a uniform background term $\alpha(\omega)$ and a $1/r$ Coulomb-like term $\beta(\omega)/r$. This is structurally identical to the potential appearing in the Schrödinger equation for hydrogen. It arises here not as a quantum postulate but as a consequence of the proton's density imprint on the vacuum. The parallel with GG Part II is direct: in the GRIN framework, the optical index $n_G(r) = 1 + 2\alpha_G R_C^{(0)}/r$ has exactly the same $1 + \beta/r$ shape.

5.5 The Dispersion Relation: The Linchpin

$$\omega = D\, q^2, \quad D = \frac{\hbar}{2m_{\text{eff}}} \tag{W.8}$$

The full Madelung/Bogoliubov dispersion (their Appendix, Eq. A.21) is:

$$\omega^2 = c_L^2\, k^2 + D^2\, k^4 \tag{W.A.21}$$

In an ordinary sound wave, frequency is proportional to wavenumber: $\omega = c_L k$. White et al. show that the quantum-pressure term in the Madelung fluid adds a k^4 correction, and in the short-wavelength limit the dispersion becomes $\omega \propto k^2$. This $\omega = Dk^2$ law is precisely the dispersion of the Schrödinger equation — but here it emerges from the classical hydrodynamics of the vacuum fluid, not as a postulate. The constant $D = \hbar/(2m_{\text{eff}})$ matches the kinetic-energy operator of quantum mechanics, but m_{eff} is the effective inertial parameter of the vacuum's longitudinal mode, identified by calibration as the electron–proton reduced mass μ.

Derivation of the Madelung Dispersion

Starting from the Schrödinger equation (White et al. Appendix, Eq. A.1), $i\hbar\partial_t\psi = -(\hbar^2/2\mu)\nabla^2\psi + V\psi$, and performing the Madelung transformation $\psi = \sqrt{\rho}\, e^{iS/\hbar}$ with hydrodynamic velocity $\mathbf{v} = \nabla S/\mu$, one obtains the continuity equation (A.4) and the Euler-like momentum equation (A.5). Linearising small perturbations $\rho = \rho_0 + \rho_1$, $\mathbf{v} = \mathbf{v}_1$ about a static equilibrium, the quantum potential $Q = -(\hbar^2/2\mu)\nabla^2\sqrt{\rho}/\sqrt{\rho}$ contributes a linearised restoring force:

$$-\nabla Q \approx \frac{\hbar^2}{4\mu\rho_0}\, \nabla(\nabla^2\rho_1). \tag{W.A.14}$$

The resulting wave equation for density perturbations is (their Eq. A.17):

$$\frac{\partial^2\rho_1}{\partial t^2} = c_L^2\, \nabla^2\rho_1 - \frac{\hbar^2}{4\mu^2}\, \nabla^4\rho_1. \tag{W.A.17}$$

Substituting a plane wave $\rho_1 \propto e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$ yields immediately:

$$\omega^2 = c_L^2 k^2 + \frac{\hbar^2}{4\mu^2}\, k^4 = c_L^2 k^2 + D^2 k^4, \quad D \equiv \frac{\hbar}{2\mu}. \tag{W.A.19–21}$$

5.6 Emergence of the Hydrogenic Spectrum

$$u''(r) + \left[\alpha(\omega) + \frac{\beta(\omega)}{r} - \frac{\ell(\ell+1)}{r^2}\right] u(r) = 0, \quad u(0) = u(\infty) = 0. \tag{W.12}$$

Choosing $\alpha(\omega_n) = -\kappa_n^2 < 0$ (reactive stop band, bound states) and $\beta(\omega_n) = \beta$ (independent of n) reduces (W.12) to the standard hydrogenic Coulomb radial equation, with quantised spatial scale:

$$\kappa_n = \frac{\beta}{2n}, \quad n = 1, 2, 3, \ldots;\ \ell = 0, 1, \ldots, n-1;\ m = -\ell, \ldots, \ell. \tag{W.eigen}$$

The temporal dispersion (W.8) then maps each spatial eigenvalue κ_n to a frequency:

$$\omega_n = D\kappa_n^2 = \frac{D\beta^2}{4} \cdot \frac{1}{n^2} \implies E_n = -\hbar\omega_n \propto -\frac{1}{n^2}. \tag{W.10}$$

The acoustic pressure field $p(r, \theta, \varphi)$ factorises into a radial part and a spherical harmonic, exactly as in hydrogen. The radial equation is mathematically identical to the Schrödinger equation for hydrogen once the acoustic operator coefficient takes the $\alpha + \beta/r$ form. Regularity conditions (no blow-up at $r = 0$, decay as $r \to \infty$) automatically impose the quantisation condition $n = 1, 2, 3, \ldots$. No Planck postulate is needed: the integer n emerges from boundary conditions, and the $\hbar$ in the dispersion constant $D = \hbar/(2\mu)$ is identified by calibration to the observed Rydberg frequency.

5.7 Calibration and Numerical Values to 7 Significant Figures

$$\mu = \frac{m_e m_p}{m_e + m_p} = 9.104426 \times 10^{-31}\ \text{kg} \tag{β-1.1}$$

$$D = \frac{\hbar}{2\mu} = 5.791535 \times 10^{-5}\ \text{m}^2\,\text{s}^{-1} \tag{β-1.2}$$

$$\omega^* = 2\pi c R_H = 2.065951 \times 10^{16}\ \text{rad}\,\text{s}^{-1} \tag{β-1.3}$$

$$a_0^{(H)} = \frac{m_e}{\mu}\, a_0^{(\infty)} = 5.291774 \times 10^{-11}\ \text{m} \tag{β-1.4}$$

$$\beta = 2/a_0^{(H)} = 3.779451 \times 10^{10}\ \text{m}^{-1} \tag{β-1.5}$$

$$\kappa_n = \frac{1}{n\, a_0^{(H)}} = \frac{1.889726 \times 10^{10}}{n}\ \text{m}^{-1} \tag{β-1.6}$$

$$\omega_n = \frac{\omega^*}{n^2} = \frac{2.065951 \times 10^{16}}{n^2}\ \text{rad}\,\text{s}^{-1} \tag{β-1.7}$$

Table 1: White et al. (2026) hydrogenic eigenfrequencies and energy levels to 7 significant figures (from their Table I, extended).

n	κ_n (m^{-1})	ω_n ($\mathrm{rad\,s}^{-1}$)	$\lvert E_n \rvert$ (eV)	f_n (PHz)	Obs. $\lvert E_n \rvert$ (eV)
1	1.889726×10^{10}	2.065951×10^{16}	13.59843	3.288051	13.59843
2	9.448628×10^{9}	5.164878×10^{15}	3.399608	0.822013	3.399608
3	6.299085×10^{9}	2.295501×10^{15}	1.510937	0.365339	1.510937
4	4.724314×10^{9}	1.291219×10^{15}	0.849902	0.205503	0.849902
5	3.779451×10^{9}	8.263805×10^{14}	0.543937	0.131522	0.543937
6	3.149543×10^{9}	5.738753×10^{14}	0.377734	0.091335	0.377734
7	2.699609×10^{9}	4.214188×10^{14}	0.277519	0.067103	0.277519

5.8 The Acoustic Vacuum and the GRIN Vacuum: Structural Comparison

Table 2: White (2026) Acoustic Vacuum ↔ GG Part II GRIN Framework: Point-by-Point Structural Mapping.

White et al. (2026) — Acoustic Vacuum	**GG Part II (March 4, 2026) — GRIN Vacuum**
Vacuum is a longitudinal compressible continuum	Vacuum is an extinction-shift GRIN optical medium
Effective sound speed $c_s(r)$ varies with position	Local light speed $c(r) = c_0/n(r)$ varies with position
$1/c_s^2(r) = A(\omega) + C(\omega)/r$	$n(r) = 1 + 2\alpha_G R_C^{(0)}/r$ (weak-field limit)
Wave equation: $(\nabla^2 + k_{\text{eff}}^2)p = 0$	GRIN ray equation: $d[n\,d\mathbf{r}/ds]/ds = \nabla n$
$k_{\text{eff}}^2 = \alpha + \beta/r$ (Coulombic)	$n(r)$ gradient force $\propto \nabla(1 + \beta/r)$
Dispersion $\omega = Dq^2$, $D = \hbar/(2\mu)$	ZBW frequency $\omega_C = c_0/R_C^{(0)}$ (intrinsic resonator)
Hydrogenic bound states from stop-band $\alpha(\omega_n) < 0$	GRIN self-trapping: photons bent back inside radian sphere
Angular labels (ℓ, m) from S^2 Laplace–Beltrami spectrum	Whittaker sinc standing wave $\propto \sin(kr)/(kr)$ as isotropic superposition
Calibration: $D = \hbar/(2\mu)$ from Rydberg frequency	Primitives: $\{\hbar, e, \omega_C\}$; c_0 is boundary condition only
Models hydrogen (one-body problem)	Models all particles; unifies EM and gravity (two Whittaker modes)

The fundamental insight shared by both frameworks is stated with precision by White et al.:

> *Quadratic dispersion,* $\omega = Dq^2$ *with* $D = \hbar/(2m_{\text{eff}})$*, is the essential ingredient that renders the dynamic-vacuum acoustic problem analytically isospectral with hydrogen.* — White et al. (2026), §V.

GG Part II's corresponding insight is that the vacuum's refractive-index gradient $\nabla n(r)$ — which is identically the gradient of the inverse acoustic speed in White et al.'s language — is the single physical source of all observable forces, particle masses, and orbital structure. The two descriptions are coordinate-system choices for the same physical vacuum: one acoustical (pressure and density), one optical (refractive index and phase speed). They are dual formulations.

5.9 Where GG Part II Goes Further than White et al. (2026)

White et al.'s (2026) paper solves the one-body hydrogen problem. It does not address:

1. The origin of the Coulombic density imprint $\rho(r) = \gamma/r^4$ — treated as an ansatz. GG Part II derives the $1/r$ GRIN index from the STAR resonator as a self-consistent standing-wave solution.

2. The unification of gravity and electromagnetism — White et al. work entirely within the electromagnetic (transverse/Coulombic) sector. GG Part II's Whittaker decomposition shows that the symmetric longitudinal sector $(F + G)$ of the same vacuum is gravity.

3. The particle as a focal point of the Russell cubic lattice.

4. Newton's G as the longitudinal-mode inverse permittivity.

5. Sommerfeld fine structure and planetary perihelion from the same GRIN Binet equation.

White et al.'s work is thus a rigorous, peer-reviewed confirmation of the acoustic-vacuum interpretation of the Coulomb problem — the sub-framework corresponding to GG Part II's electromagnetic GRIN sector.

6 The Physical Foundation: Re-emission at the Local Speed of Light

Every photon propagating through any medium — material or vacuum — is absorbed and re-emitted by the constituent oscillators of that medium. Each re-emission produces a new photon at speed c relative to the local emitter at that point. In the vacuum near a mass, those emitters are the polarised zero-point vacuum oscillators, whose permittivity $\varepsilon(r)$ and permeability $\mu(r)$ differ from free-space values. The resulting phase speed at position **r** is:

$$c(r) = \frac{c_0}{\sqrt{\varepsilon(r)\mu(r)}} = \frac{c_0}{n(r)} \tag{26}$$

where $n(r)$ is the local refractive index of the polarised vacuum and c_0 is the asymptotic far-field value — a boundary condition, not a universal constant. This mechanism is plain optics. It requires no Lorentz transformation, no spacetime curvature, and no special relativity.

Light does not travel through empty, inert space. Near any mass, the vacuum itself is slightly altered — squeezed, polarised — like a lens material. The denser the optical squeeze at a given point (captured by the number $n(r)$), the slower light travels there. Farther from any mass, $n \to 1$ and light returns to its maximum speed c_0. This one idea — the speed of light varies with position — is the entire engine of this paper. Gravity, orbits, and particle masses all follow from it.

The extinction-shift mechanism defines a smooth function $n : M \to \mathbb{R}_{>0}$. This function is a *section* of the trivial positive-real line bundle over M. The connection induced by $n(r)$ on this bundle is the 1-form

$$\omega_n = \frac{dn}{n} = d(\ln n), \tag{27}$$

which is exact (since $M = \mathbb{R}^3$ is simply connected: $H^1(\mathbb{R}^3) = 0$). An exact connection has zero holonomy around every loop — there are no topological trapping effects from n alone. The topological non-triviality required for EM (winding number, holonomy, charge) must come from a separate $U(1)$ bundle, as established in Section 3.
This is precise: the GRIN index $n(r)$ accounts for *magnitude* (gravitational and mass effects), while the $U(1)$ bundle accounts for *phase* (charge, Aharonov–Bohm).

Lorentz-Variant Framework by Construction. This framework is Lorentz-variant by construction. The preferred frame at every point in space is the rest frame of the local vacuum medium. Lorentz invariance is an asymptotic property, valid only where $n(r) \to 1$. It is not a fundamental symmetry; it is a far-field approximation.

7 Russell's Cubic Wave-Fields: The Cosmological Setting

7.1 The Universe as Nested Optical Cavities

Walter Russell described the universe as a system of nested wave-fields, each bounded by three mutually perpendicular planes of zero curvature — the planes of magnetic stillness. Within each cubic wave-field, centripetal motion increases density toward the gravitational focus at the cube's centre. At the boundary planes, density reaches its minimum and the lattice's restoring force changes sign.
In extinction-shift GRIN language: each cubic wave-field is a region of vacuum whose refractive index $n(r)$ rises from near unity at the cube's face toward a maximum at the gravitational focus. The planes of magnetic stillness are the saddle surfaces of the composite GRIN field where $\nabla n = 0$.

The Russell cubic lattice defines a CW-complex X whose 3-cells are the cubic wave-fields, 2-cells are the shared faces, 1-cells are the edges, and 0-cells are the vertices. For the infinite cubic lattice:

$$H_0(X;\mathbb{Z}) = \mathbb{Z}, \quad H_1(X;\mathbb{Z}) = 0, \quad H_2(X;\mathbb{Z}) = 0, \quad \ldots \tag{28}$$

(the lattice is contractible as a topological space — all higher homology vanishes). The topological non-triviality is *not* in the lattice itself but in the fiber bundles supported over it: the $U(1)$ EM bundle becomes non-trivial around the particle-formation focal points, where the winding number of the connection 1-form is ± 1. Each focal point is a *defect* in the $U(1)$ bundle — a non-removable singularity of $\mathcal{A}$ that corresponds to electric charge.

7.2 The Saddle Surface and the Biconvex Lens

Consider two adjacent cubic wave-fields, centred on masses M_A and M_B separated by lattice vector $\mathbf{d}$. Each mass produces a GRIN field:

$$n_A(\mathbf{r}) = 1 + \frac{2\alpha_G R_C^{(A)}}{|\mathbf{r} - \mathbf{r}_A|}, \qquad n_B(\mathbf{r}) = 1 + \frac{2\alpha_G R_C^{(B)}}{|\mathbf{r} - \mathbf{r}_B|}. \tag{29}$$

Each mass thickens the optical medium of the vacuum around it in an ever-weakening halo. The halos follow a $1/r$ law — exactly like gravity, which is no coincidence. The shared face region has its lowest refractive index at the centre and higher index at its edges — the precise optical structure of a biconvex converging lens for incoming wavefronts.

8 Particle Formation: Two Cones of Light Converging to a Common Point

8.1 The Photon as a Longitudinal Compression Wave

This subsection addresses the heart of the framework's physical claim, which is the most counter-intuitive for readers trained in post-Heaviside electrodynamics.

A free photon propagating through the extinction-shift GRIN vacuum is, at the level of the vacuum medium, a **longitudinal compression wave** — a pressure disturbance in the GRIN medium, exactly analogous to a sound wave. It is described by a *pure-scalar* quaternion:

$$q_{\text{photon}} = \psi_0 \, e^{i(kz-\omega t)} + \mathbf{0}, \tag{30}$$

where ψ_0 is the wave amplitude and the pure-quaternion part $\text{Pu}(q) = \mathbf{0}$ vanishes — there is no transverse circulation in a free, undisturbed photon in the GRIN medium.

This claim sounds strange because we are taught that light is a transverse wave. But "transverse" refers to the direction in which the *electric field vector* oscillates relative to propagation. This is a far-field description of the *interaction* of light with matter. At the vacuum level — in the extinction-shift picture where the vacuum itself is the medium — the photon is a longitudinal pressure pulse in the GRIN medium. This is exactly what White et al. (2026) model: the vacuum is an acoustic continuum, and its modes are longitudinal pressure waves. When the photon interacts (is absorbed and re-emitted), the transverse components appear as the angular structure of the dipole re-emission pattern. The transverse character of light is a property of the *interaction*, not of the vacuum mode itself.

How Two Transverse Components Emerge from Two Longitudinal Photons

When two dipole photons travel toward each other along the $\pm\hat{z}$ axis, each carries its longitudinal phase wave plus the residual transverse angular momentum of its dipole emission pattern (helicity). The full quaternion of each incoming photon is:

$$q_+ = \underbrace{\psi_0}_{\text{scalar: long.}} + \underbrace{\mathbf{k}\,|p|}_{\text{pure: helicity}}, \qquad q_- = \underbrace{-\psi_0}_{\text{time-conjugate}} + \underbrace{\epsilon\,\mathbf{k}\,|p|}_{\text{pure: helicity}}, \tag{31}$$

where $\epsilon = +1$ for parallel helicity (Case A) and $\epsilon = -1$ for anti-parallel helicity (Case B). The sign reversal $-\psi_0$ in q_- arises because the two wavefronts are time-conjugates: they emanate from opposite cubes and meet at the focal point with a definite phase relationship (see Section 8).

Case B: Anti-parallel helicity ($\epsilon = -1,\ J_z = 0$).

$$q_+ + q_- = (\psi_0 - \psi_0) + (\mathbf{k}|p| - \mathbf{k}|p|) = 0. \tag{32}$$

Complete cancellation. No standing wave forms. This is the channel for neutral, spinless bosons (scalar excitations).

Case A: Parallel helicity ($\epsilon = +1,\ J_z = +2\hbar$). The product of the two quaternions is:

$$\begin{aligned} q_+\,\bar{q}_- &= (\psi_0 + \mathbf{k}|p|)\,\overline{(-\psi_0 + \mathbf{k}|p|)} \\ &= (\psi_0 + \mathbf{k}|p|)\,(-\psi_0 - \mathbf{k}|p|) \\ &= -\psi_0^2 - \psi_0\mathbf{k}|p| - \psi_0\mathbf{k}|p| - (\mathbf{k}|p|)^2 \\ &= -\psi_0^2 + |p|^2 - 2\psi_0|p|\,\mathbf{k}. \end{aligned} \tag{33}$$

Separating scalar and pure parts:

$$\underbrace{\mathrm{Sc}(q_+\bar{q}_-)}_{\text{longitudinal mode}} = -\psi_0^2 + |p|^2 = |p|^2 - \psi_0^2 \longrightarrow \underbrace{(F+G)\text{-sector}}_{\textbf{mass / gravity}} \tag{34}$$

$$\underbrace{\mathrm{Pu}(q_+\bar{q}_-)}_{\text{transverse mode}} = -2\psi_0|p|\,\mathbf{k} \longrightarrow \underbrace{(F-G)\text{-sector}}_{\textbf{charge / EM}} \tag{35}$$

From the collision of two longitudinal photon waves (each described by $q = \psi_0 + \mathbf{k}|p|$), the quaternion product yields:

- **Scalar part** $= |p|^2 - \psi_0^2$: the Lorentz invariant of the collision — on-shell, $|p|^2 = \psi_0^2/c^2$, so the scalar part is the rest-mass term. This is the **longitudinal compression** stored in the standing wave: it is the $(F+G)$ Whittaker mode, the **gravitational / mass sector**.

- **Pure-quaternion part** $= -2\psi_0|p|\,\mathbf{k}$: the surviving **residual transverse circulation** along $\hat{z}$. This is non-zero *only because the two photons carried identical helicity* — if the helicities cancelled, this term would be zero. This is the $(F-G)$ Whittaker mode, the **electromagnetic / charge sector**.

The two transverse components $(F \pm G)$ are not assumed: they emerge as the scalar and pure-quaternion parts of the quaternion product of two longitudinal photon waves with matching helicity.

Think of two spinning tops colliding head-on, both spinning in the same direction. Before the collision, each top is a pure forward-moving object (longitudinal). After the collision, the spinning motions add: the combined object has both a compressed body (the mass — the longitudinal part) and a net rotation (the charge — the transverse part). If the two tops spun in *opposite* directions before the collision, the rotations would cancel and the result would be a pure compression with no rotation (neutral particle, no charge). This is exactly Cases A and B above, in mechanical language. The "two transverse components" are therefore the symmetric component of the spin (which becomes mass, via the $(F+G)$ mode) and the antisymmetric residual rotation (which becomes charge, via the $(F-G)$ mode). Both emerge from the interaction of two longitudinal waves — neither exists in the individual free photon before the collision. This is why a free photon has no rest mass and no charge: both are products of the interaction, not properties of the individual wave.

Before the collision, each free photon propagates in the trivial scalar sector: $\pi_1 = 0$, no winding, no charge. The collision in Case A produces a standing-wave configuration whose pure-quaternion part $-2\psi_0|p|\,\mathbf{k}$ winds around the $\hat{z}$ axis with winding number $+1$ (from the $(2,1)$ torus-knot topology derived in Part IV from the $J_z = 2\hbar$ angular momentum budget). The transition from $\pi_1 = 0$ (free photon) to $\pi_1 = \mathbb{Z}_1$ (charged particle) is a genuine *topological phase transition* induced by the collision. Before: trivial $\mathbb{R}$-bundle. After: non-trivial $U(1)$-bundle. The first Chern class jumps from $c_1 = 0$ to $c_1 = 1$. This is the creation of one unit of electric charge from two uncharged photons.

8.2 The Cone Geometry

A photon wavefront expanding outward from the gravitational focus of cube A is a diverging spherical wavefront. As it enters the biconvex GRIN lens region at the cube's shared face, extinction-shift re-emission at each successive vacuum oscillator deflects the photon's trajectory toward the region of higher n. The net effect converts the diverging spherical wave into a converging conical envelope — a cone of photon trajectories all angled inward toward a common focal point on the far side of the lens, inside cube B.

Each photon is described by its propagation quaternion

$$q_{\text{photon}}(t) = \phi(t) + \mathbf{i}p_x(t) + \mathbf{j}p_y(t) + \mathbf{k}p_z(t), \tag{36}$$

where $\phi(t)$ is the wave phase and $\mathbf{p} = (p_x, p_y, p_z)$ is the momentum direction. A photon propagating along $+\hat{z}$ has $q_+ = \phi + \mathbf{k}|\mathbf{p}|$. Its time-conjugate propagating along $-\hat{z}$ has $q_- = \phi - \mathbf{k}|\mathbf{p}|$.

The quaternion product of the two counter-propagating wavefronts is:

$$q_+\,\bar{q}_- = (\phi + \mathbf{k}|\mathbf{p}|)(\phi + \mathbf{k}|\mathbf{p}|) = \phi^2 - |\mathbf{p}|^2 + 2\phi|\mathbf{p}|\mathbf{k}. \tag{37}$$

The scalar part $\phi^2 - |\mathbf{p}|^2$ is the Lorentz invariant (longitudinal/mass sector). The pure part $2\phi|\mathbf{p}|\mathbf{k}$ is the residual circulation (transverse/charge sector). The standing wave formed at the focal point *inherits both parts*: a longitudinal compression (gravity, mass) and a residual transverse circulation (charge).

8.3 Superposition Produces the Particle

The geometry is symmetric. From cube A, a spherical wavefront converges through the lens into cube B as a cone. From cube B, an identical wavefront passes through the same lens in the opposite direction. The two cones are counter-propagating, and where both reach focus simultaneously, their waves add constructively into a standing wave:

$$\Phi_A + \Phi_B \longrightarrow \Phi_0 \frac{\sin k(r)\, r}{k(r)\, r} \cos(\omega_C t) \qquad \text{(sinc standing wave at focal point).} \tag{38}$$

Two waves arriving from opposite directions, at the same frequency, create a standing wave — like two piano strings vibrating against each other produce a note that hangs in the air. The mathematical shape of this standing wave is a sinc function: a sharp central peak surrounded by diminishing ripples. This peaked, stationary pattern of electromagnetic energy is the particle — not a tiny ball, but a standing wave of light, trapped at the focal point of the universe's cubic optical geometry.

8.4 The Self-Reinforcing Optical Trap

Once the sinc standing wave exists at the focal point, it creates its own GRIN field — the $n(r)$ polarisation of the surrounding vacuum. This deepens the gravitational well at the focal point, which makes the biconvex lens more convergent, which focuses incoming wavefronts more tightly, which reinforces the sinc amplitude. The system is a self-sustaining optical cavity: the particle creates the lens, and the lens creates the particle.

The self-trapping condition is a fixed-point equation for the refractive-index profile:

$$n[r;\, \Phi(r)] = n[r;\, \Phi(r)], \tag{39}$$

where $\Phi(r)$ is the sinc field amplitude and n is the refractive index induced by Φ. By the Banach fixed-point theorem, this has a unique stable solution in the appropriate function space (provided the GRIN coupling is below the Planck threshold of Section 18). The fixed point is the *particle*.

Topologically, the fixed-point attractor is a stable manifold in the infinite-dimensional space of field configurations. The basin of attraction is the set of initial field configurations that flow, under the self-trapping dynamics, to the STAR resonator.

9 The STAR Resonator: Sinc Standing Wave as Particle

9.1 The Local Radian Sphere

The sinc standing wave has a natural boundary: where the oscillatory near-field transitions to smooth far-field behaviour, at the point where $k(r) \cdot R_C = 1$:

$$R_C(r) = \frac{c(r)}{\omega_C} \qquad \text{(local Compton radius).} \tag{40}$$

The particle has a characteristic size — the radius of the innermost ripple of its sinc standing wave. Inside, the field oscillates wildly; outside, it fades smoothly as a gentle $1/r$ halo. The size of this skin depends on where the particle is: deep inside a gravitational well (where c is smaller), the particle is physically more compact.

9.2 The GRIN Self-Trapping Condition: Fixing the Compton Radius

$$R_C^{(0)} = \frac{c_0}{\omega_C} = \frac{\hbar}{m_e^{(0)} c_0} = 3.861594 \times 10^{-13}\ \text{m} \tag{41}$$

At the particle's natural skin radius, photons trying to escape are bent back inward by the refractive-index halo the particle itself creates. The particle is like a lighthouse whose light bends back into the building — perpetually self-confined. The number 3.862×10^{-13} m is the electron's reduced Compton wavelength, a well-measured quantity. Here it emerges not from quantum mechanics but purely from the geometry of GRIN optics.

9.3 Mass and Inertia as Stored Field Energy

$$U_{\text{res}} = \hbar\omega_C \qquad \text{(position-independent: frequency is conserved through GRIN)} \tag{42}$$

$$m(r) = \frac{\hbar\omega_C}{c(r)^2} = m_e^{(0)} \cdot n(r)^2 \tag{43}$$

Mass is nothing more than stored energy per unit c^2. Since the stored energy $\hbar\omega_C$ stays fixed but the local speed of light $c(r)$ decreases inside a gravitational well, the mass $m = E/c^2$ increases there.

In quaternion language, the full energy-momentum of the STAR resonator is

$$P_{\text{res}} = \underbrace{mc^2}_{\text{scalar: mass}} + \mathbf{i}\,p_x + \mathbf{j}\,p_y + \mathbf{k}\,p_z, \tag{44}$$

where p_i are the three momentum components. At rest ($\mathbf{p} = 0$), the quaternion is *pure scalar*: $P_{\text{res}} = mc^2$. This is the longitudinal mode dominance: the resting STAR resonator has all its energy in the scalar (Tamas/mass) sector of $\mathbb{H}$. Motion populates the pure-quaternion part. The mass-energy relation $E = mc^2$ is therefore the statement that a stationary particle is a *pure-scalar quaternion.*

10 The GRIN Vacuum: Two Fields from One Resonator

10.1 The Two GRIN Indices

The STAR resonator generates two distinct optical halos simultaneously, because it carries two types of charge: electric charge e (coupling to the transverse Whittaker mode) and mass m_e (coupling to the longitudinal mode):

$$n_{\text{EM}}(r) = 1 + \frac{2\alpha_{\text{EM}} R_C^{(0)}}{r} \tag{45}$$

$$n_G(r) = 1 + \frac{2\alpha_G R_C^{(0)}}{r} \tag{46}$$

The electric field thickens the optical vacuum around it according to (45). The gravitational field does exactly the same thing — but $\alpha_G \approx 10^{-45}$ is forty-three orders of magnitude smaller than $\alpha_{\text{EM}} \approx 1/137$. The two formulas are identical in shape; only the coupling number differs. Gravity is not a different kind of force from electromagnetism — it is the same optical thickening mechanism, operating at an incomparably smaller amplitude.

The electromagnetic coupling α_{EM} and gravitational coupling α_G are both dimensionless. Their vast ratio

$$\frac{\alpha_{\text{EM}}}{\alpha_G} = \frac{e^2/(\hbar c_0)}{Gm_e^2/(\hbar c_0)} = \frac{e^2}{Gm_e^2} \approx 10^{43} \tag{47}$$

reflects the different topological weights of the two sectors. In the $U(1)$ bundle, the EM mode couples via the *winding number* $n \in \mathbb{Z}$ (a topological invariant of the fiber S^1), which is necessarily an integer and of order 1. The gravitational mode couples via a *metric deformation* with no topological quantisation — the coupling $\alpha_G = Gm_e^2/(\hbar c_0)$ is a continuous, unquantised number. The hierarchy is a direct consequence of the topological disparity between a discrete $\mathbb{Z}$-charge and a continuous real-valued coupling.

10.2 Gravitational Acceleration as an Extinction-Shift Gradient Force

$$\mathbf{g}(r) = -c(r)^2\,\nabla \ln n_G(r) \approx +\frac{2\alpha_G c_0^2 R_C^{(0)}}{r^2}\,\hat{r}_{\text{inward}} \tag{48}$$

Gravity is simply the pressure of the optical gradient — photons (and everything else) are constantly nudged toward the thicker part of the optical vacuum, meaning toward masses. You feel gravity pulling you toward Earth for the same reason a prism bends light toward its thicker edge.

11 Gravity as Frequency Synchronisation

11.1 The GRIN Redshift of a Coupled Resonator

$$\omega_{\text{obs}}(r) = \frac{\omega_C}{n_G(r)} \approx \omega_C\left(1 - \frac{Gm_1}{c(r)^2 r}\right) \tag{49}$$

11.2 Gravity as Synchronisation Energy

$$\Delta E(r) = \hbar\,\Delta\omega(r) = \hbar\omega_C \cdot \frac{2Gm_1}{c(r)^2 r} \tag{50}$$

$$F_{\text{grav}}(r) = -\frac{d(\Delta E)}{dr} = -\frac{Gm_1 m_2}{r^2} \qquad \text{(Newton's law, far field)} \tag{51}$$

Gravity is the force that drives two coupled mass oscillators toward frequency synchronisation through the extinction-shift re-emission chain connecting them.

In fiber-bundle language, gravitational attraction is *parallel transport* of the resonator's phase along the connection $\mathcal{A}_G$. Two particles at different radii have their internal clocks (frequencies $\omega_C/n_G(r)$) running at different rates. The force (51) is the gradient of the holonomy discrepancy:

$$F_{\text{grav}} = -\nabla(\hbar\,\Delta\omega_{\text{holonomy}})\,, \quad \Delta\omega_{\text{holonomy}} = \omega_C\left[\frac{1}{n_G(r_1)} - \frac{1}{n_G(r_2)}\right]. \tag{52}$$

Gravity is the restoring force that minimises the holonomy mismatch between two gravitationally coupled STAR resonators.

12 Dowdye's Six Extinction-Shift Equations in Local-c Form

Dowdye's six equations constitute a complete description of how the extinction-shift mechanism governs mass, time, frequency, energy, and angular deflection in the locally-varying-c vacuum. All six must be read with locally-varying $c(r)$, not the far-field constant c_0.

$$m_{\text{eff}} = \frac{m_0}{\sqrt{1 - v^2/c(r)^2}} \qquad \text{(D.1 Effective mass)}$$

$$\tau_{\text{tr}} = \frac{\tau_0}{\sqrt{1 - v^2/c(r)^2}} \qquad \text{(D.2 Transit time)}$$

$$\Delta v = v_0 \cdot \frac{GM}{R\,c(r)^2} \qquad \text{(D.3 Velocity shift)}$$

$$E = m_0\,c(r)^2 \qquad \text{(D.4 Rest energy (local))}$$

$$\Delta\omega = \frac{3\omega GM}{a(1-e)\,c(r)^2} \qquad \text{(D.5 Orbital frequency shift)}$$

$$\delta\theta = \frac{4GM}{R\,c(r)^2} \qquad \text{(D.6 Angular deflection)}$$

12.1 The Dual-Cone Focus as a Dowdye Summation

A photon wavefront from cube A travels through the shared-face lens region. Integrating the velocity-shift equation (D.3) over the full lens transit gives a cone half-angle of approximately $\delta\theta_{\text{cone}}/2 = 2GM_A/(d\,c_0^2)$, where d is the lattice spacing. For two equal adjacent masses, this places the focal point near the cube's centre, consistent with the Russell picture.

13 The GRIN Master Equation: Fermat's Principle for Orbital Mechanics

13.1 Fermat's Principle in the Extinction-Shift Vacuum

$$\delta \int n(r)\,ds = 0 \implies \frac{d}{ds}\left[n(r)\,\frac{d\mathbf{r}}{ds}\right] = \nabla n(r) \qquad (53)$$

Fermat's principle says light — and, in this framework, everything — takes the path through space that minimises the total optical effort. In a variable-index medium, this path is not a straight line: it curves toward denser optical regions. This single equation governs both the bending of light around the Sun and the precession of Mercury's orbit.

Equation (53) is the geodesic equation for the conformal metric

$$ds^2_{\text{opt}} = n(r)^2 \left(dx^2 + dy^2 + dz^2\right). \tag{54}$$

This is an *optical metric* on $M = \mathbb{R}^3$, conformally related to the flat metric. The GRIN ray trajectories are geodesics of g_{opt}. In topological terms: the conformal factor $n(r)$ deforms the metric without changing the topology of $M = \mathbb{R}^3$, which remains simply connected. The topological non-triviality (charge, winding) lives entirely in the fiber bundle over M, not in M itself.
The correspondence with GR is instructive:

Framework	Curved space	Source of curvature
GR	Spacetime metric $g_{\mu\nu}$	Stress-energy tensor $T_{\mu\nu}$
GRIN	Optical metric $n(r)^2\delta_{ij}$	Refractive index gradient ∇n

13.2 The GRIN Binet Equation

Substituting $u = 1/r$ and the weak-field GRIN form $n(r) = 1 + \beta/r$ into (53), the orbital equation becomes the Binet equation with a GRIN correction term:

$$\frac{d^2u}{d\varphi^2} + u = \frac{1}{L_0^2} + \frac{3\beta}{L_0^2}\,u + O(\beta^2) \tag{55}$$

$$\text{Electromagnetic orbit } (n_{\text{EM}}): \quad \beta_{\text{EM}} = 2\alpha_{\text{EM}} R_C^{(0)} = 2r_e = 5.635883 \times 10^{-15}\ \text{m} \tag{56}$$

$$\text{Gravitational orbit } (n_G): \quad \beta_G = 2\alpha_G R_C^{(0)} = 2Gm_e/c_0^2 = 1.352955 \times 10^{-57}\ \text{m} \tag{57}$$

13.3 The Secular Perihelion Advance

$$\delta\phi = \frac{3\pi\beta}{a(1-e^2)} \tag{58}$$

This single formula predicts three distinct phenomena: with β_G and Mercury's orbit $\to$ 42.96″/century of perihelion precession; with β_{EM} and the Bohr orbit $\to$ Sommerfeld fine structure; with β_G and the electron's ZBW orbit $\to$ the electron's gravitational self-precession (predicted, not yet measured).

14 The Sommerfeld–GRIN Bridge: Fine Structure without Special Relativity

Sommerfeld's 1916 derivation used special relativity as an essential input. In this framework, the same formula is derived from the GRIN ray equation with no special relativity. The relativistic mass increase and the GRIN ray-bending effect are mathematically identical to first order — but their physical origins are completely different.

$$\delta\phi_{\mathrm{EM}} = \frac{3\pi\beta_{\mathrm{EM}}}{a_0} = \frac{3\pi \cdot 2\alpha_{\mathrm{EM}} R_C^{(0)}}{R_C^{(0)}/\alpha_{\mathrm{EM}}} = 6\pi\alpha_{\mathrm{EM}}^2 = 6\pi \times (7.297353 \times 10^{-3})^2 = 1.003765 \times 10^{-3} \text{ rad/orbit} \tag{59}$$

The fine-structure result $\delta\phi = 6\pi\alpha_{\mathrm{EM}}^2$ is a *closure failure*: the electromagnetic orbit in the GRIN field does not close exactly. In topological terms, this is the holonomy of the $U(1)$ connection $\mathcal{A}_{\mathrm{EM}}$ around the Bohr orbit γ_{Bohr}:

$$\mathrm{Hol}_{\gamma_{\mathrm{Bohr}}}(\mathcal{A}_{\mathrm{EM}}) = e^{i\,\delta\phi_{\mathrm{EM}}} = e^{6\pi i\,\alpha_{\mathrm{EM}}^2} \neq 1. \tag{60}$$

The orbit fails to close because the $U(1)$ holonomy is not trivial: the electron's wave function acquires a phase $6\pi\alpha_{\mathrm{EM}}^2$ per orbit. This is the GRIN analogue of the Aharonov–Bohm phase, with the role of the magnetic flux played by the Compton-scale curvature of the EM GRIN field. The fine-structure constant α_{EM} is thus a *topological invariant* — the square root of the holonomy phase per orbit divided by 6π.

The GRIN correction to the Binet equation (55) at order β reads, in quaternion language, as a precession of the Runge–Lenz quaternion $\mathbf{A} = \mathbf{p} \times \mathbf{L} - me^2\hat{r}$ (the axis of the orbit):

$$\frac{d\mathbf{A}}{dt} = \frac{3\beta\,\omega_C}{r}\,(\mathbf{A} \times \hat{r}). \tag{61}$$

Integrating over one orbit and using $|\mathbf{A}| = me^2\sqrt{1-\alpha_{\mathrm{EM}}^2}$, the precession angle per orbit is

$$\delta\phi = \frac{|\delta\mathbf{A}|}{|\mathbf{A}|} = 6\pi\alpha_{\mathrm{EM}}^2, \tag{62}$$

which recovers (59). The quaternion description makes explicit that fine structure is a *precession of the orbital plane* — a rotation in $\mathbb{R}^3$ — not a relativistic mass effect.

15 The Electron's Gravitational Perihelion and Zitterbewegung

Applying the same perihelion formula to the electron's own ZBW orbit at radius $a = R_C^{(0)}$:

$$\delta\phi_{\mathrm{ZBW}} = \frac{3\pi\beta_G}{R_C^{(0)}} = 6\pi\alpha_G = 6\pi \times 1.751810 \times 10^{-45} = 3.302083 \times 10^{-44}\ \mathrm{rad/orbit} \quad (63)$$

This is a pure prediction of the framework, awaiting future detection.

16 Whittaker's Decomposition: Two Forces from One Wave Equation

16.1 The Sinc Standing Wave is the Whittaker Isotropic Scalar Potential

Whittaker (1903) proved that any solution to the scalar wave equation $\nabla^2\Phi = c^{-2}\ddot{\Phi}$ can be written as an angular integral of plane waves. The isotropic case reduces exactly to:

$$\Phi_W(r,t) = \Phi_0 \frac{\sin(kr)}{kr} \cos(\omega t) \quad (64)$$

This is the mathematical shape of the STAR resonator: a central peak that oscillates in time, surrounded by concentric ripple shells that diminish outward. Whittaker proved in 1903 that this exact shape is what you get when waves arrive equally from every direction in space — which is precisely what happens at the Russell cubic lattice's focal point.

16.2 The Two Polarisation Sectors

Whittaker (1904) showed that all electromagnetic fields are generated by exactly two scalar functions F and G, both satisfying the wave equation:

$$\underbrace{(F+G)}_{\text{symmetric}} \longrightarrow \text{longitudinal mode} \qquad \underbrace{(F-G)}_{\text{antisymmetric}} \longrightarrow \text{transverse mode} \quad (65)$$

Let $\Omega^\bullet(M)$ denote the de Rham complex of differential forms on $M = \mathbb{R}^3$. The Whittaker potentials F and G are 0-forms (scalar functions). The split $(F \pm G)$ corresponds to the decomposition of the space of scalar potentials into its $\mathbb{Z}_2$-even and $\mathbb{Z}_2$-odd parts under the time-reversal operator T:

$$(F+G) \in \Omega^0_+(M) \quad (\text{time-reversal even — } \textbf{real/longitudinal}) \quad (66)$$
$$(F-G) \in \Omega^0_-(M) \quad (\text{time-reversal odd — } \textbf{imaginary/transverse}) \quad (67)$$

The longitudinal sector Ω^0_+ is the space of T-invariant scalar functions: it is closed under d (exterior derivative) and contributes to $H^0_{\text{de Rham}}(M) = \mathbb{R}$ — a single connected component, no winding. The transverse sector Ω^0_- is the space of T-odd scalar potentials, whose curls generate the magnetic field. Its non-trivial topology resides in $H^1(M;\mathbb{Z})$ — the first cohomology, which classifies $U(1)$ bundles.

Summary:

$$\boxed{\begin{array}{l} (F+G)\text{-sector}: \;\; H^0_{\text{dR}}(M) = \mathbb{R}, \quad \pi_1 = 0, \quad c_1 = 0 \;\Rightarrow\; \textbf{gravity} \\ (F-G)\text{-sector}: \;\; H^1(M;\mathbb{Z}) = \mathbb{Z}, \quad \pi_1 = \mathbb{Z}, \quad c_1 \in \mathbb{Z} \;\Rightarrow\; \textbf{electromagnetism} \end{array}}$$

Define the quaternion field $\mathbf{Q}_W = F + \mathbf{i}\,\partial_x G + \mathbf{j}\,\partial_y G + \mathbf{k}\,\partial_z G$. Then:

$$\mathbf{Q}_W + \bar{\mathbf{Q}}_W = 2F \;=\; 2(F+G)\big|_{G=0} \quad \text{[scalar part } \times 2 \text{ — longitudinal]} \tag{68}$$

$$\mathbf{Q}_W - \bar{\mathbf{Q}}_W = 2\nabla G \quad \text{[pure-quaternion part } \times 2 \text{ — transverse]} \tag{69}$$

The Whittaker split is literally quaternion conjugation: $(F+G)$ is the *self-conjugate* (Hermitian) part of $\mathbf{Q}_W$, and $(F-G)$ is the *anti-self-conjugate* (anti-Hermitian) part. This is the canonical decomposition

$$\mathbf{Q}_W = \underbrace{\frac{\mathbf{Q}_W + \bar{\mathbf{Q}}_W}{2}}_{\text{self-conjugate (gravity)}} + \underbrace{\frac{\mathbf{Q}_W - \bar{\mathbf{Q}}_W}{2}}_{\text{anti-self-conjugate (EM)}} . \tag{70}$$

Electromagnetism and gravity are not two different forces. They are the two irreducible symmetry classes — antisymmetric and symmetric — of the same scalar wave equation in the extinction-shift GRIN vacuum. Heaviside's gauge truncation of Maxwell's original 20-equation system discarded the $(F+G)$ sector by setting the longitudinal potential to zero as pure gauge.

In this framework: that truncation was *incorrect*; the longitudinal scalar mode is gravity; and Heaviside's simplification severed the mathematical unity of the two forces.

On Objection 4 (§1.4): The Two Regimes

The $(F-G)$ transverse mode produces *both* far-field transverse radiation (photons, EM waves) *and* the near-field Coulomb binding. The $(F+G)$ longitudinal mode is the near-field *scalar binding potential* — the gravitational mass mode — not a propagating transverse radiation wave. LIGO detects the transverse *radiation* from accelerating masses; the $(F+G)$ mode is the static near-field that generates the $1/r^2$ Newtonian force. The objection correctly identifies the *radiation* modes as transverse — but it incorrectly applies this to the *binding* modes. A Coulomb electric field is also not a transverse propagating wave; it is a static longitudinal field — yet no one questions that it is real. The gravitational $(F+G)$ mode is in the same logical category.

The error was not arbitrary — it was *topologically motivated*: in flat Minkowski space with trivial topology ($M = \mathbb{R}^{3,1}$, $H^1 = 0$), the longitudinal mode carries no winding number, no Chern class, no physical charge — so setting it to zero was harmless for all *EM* computations. What was not recognised was that the longitudinal mode, though topologically trivial, carries *energy* — the gravitational energy $U_G = \alpha_G \hbar \omega_C$ — and that setting it to zero therefore eliminated gravity from the electromagnetic theory.

16.3 The Gravitational Permittivity

$$\varepsilon_G = \frac{1}{4\pi G} = 1.192297 \times 10^{9}\ \mathrm{kg\,s^2\,m^{-3}} \tag{71}$$

exactly analogous to

$$\varepsilon_0 = \frac{1}{4\pi k_e} = 8.854188 \times 10^{-12}\ \mathrm{F\,m^{-1}} \tag{72}$$

Newton's G is not a fundamental mystery — it is the permittivity of the vacuum for the longitudinal wave mode, exactly as ε_0 is the permittivity for electromagnetic waves.

In the $U(1)$ electromagnetic bundle, the permittivity ε_0 is the *fiber metric*: it determines how much energy is stored per unit square of the $U(1)$ connection strength. Formally:

$$U_{\mathrm{EM}} = \frac{\varepsilon_0}{2} \int_M |F|^2 d^3x, \quad F = d\mathcal{A}. \tag{73}$$

In the trivial $\mathbb{R}$ bundle of gravity, $\varepsilon_G = 1/(4\pi G)$ plays the identical role for the gravitational connection:

$$U_G = \frac{\varepsilon_G}{2}\int_M |\nabla\phi_G|^2 d^3x. \tag{74}$$

The two permittivities are metrics on their respective fibers — one on S^1 (electromagnetic), one on $\mathbb{R}$ (gravitational). Their enormous ratio $\varepsilon_G/\varepsilon_0 = c_0^2/(G \cdot 4\pi\varepsilon_0)$ encodes the hierarchy between the two forces.

16.4 Reed's Mass Dipole as a Whittaker Longitudinal-Mode Device

The concept of exploiting the $(F + G)$ longitudinal Whittaker sector for propulsion purposes has been developed by Larry J. Reed in *Quantum Wave Mechanics* (4th ed., 2022), Chapter 52 (Anti-Gravity). Reed's Figure 52-39 shows a notional mass-dipole gravitational drive concept in which positive mass (high-pass left-hand LC filter metamaterial) is coupled to effective negative mass (low-pass right-hand LC filter metamaterial).

In GG Part II's language, Reed's mass dipole is a device for generating an asymmetric ∇n_G — a spatial gradient in the gravitational GRIN index — without a large, symmetric gravitational mass as the source. The metamaterial structure emulates a $1/r$ GRIN profile over a finite volume by periodically varying the effective vacuum density at the sub-wavelength scale, using LC filter transmission lines whose impedance is engineered to match specific values of the local permittivity ε_G and permeability μ_G of the longitudinal Whittaker mode.

Reed's metamaterial device achieves a *local modification of the fiber metric* $\varepsilon_G(r)$. In the trivial $\mathbb{R}$ bundle of gravity, any smooth deformation of ε_G is topologically permitted (since $\pi_0(\mathcal{C}_G) = 0$: all gravitational configurations are path-connected). There is no topological obstruction to building a gravitational gradient — unlike electromagnetism, where the $\mathbb{Z}$-valued charge quantisation imposes discrete jumps. This is why an anti-gravity device is topologically conceivable for the longitudinal mode but would be impossible for the transverse $U(1)$ mode (you cannot continuously deform away a unit charge).

17 The Mode-Energy Partition and the Derivation of G

17.1 Energy in Each Whittaker Mode

$$U_{\text{EM}} = m_e^{(0)} c_0^2 = \hbar\omega_C \qquad \text{(transverse mode — all of the rest mass)} \tag{75}$$

$$U_G = \alpha_G \cdot m_e^{(0)} c_0^2 = \alpha_G \hbar\omega_C \approx 1.430 \times 10^{-78}\ \text{J} \qquad \text{(longitudinal mode)} \tag{76}$$

17.2 Newton's G — A Structural Decomposition

On Objection 3 (§1.3): Dimensional Consistency

The structural claim is not that $G = 1/\varepsilon_0$ (dimensionally impossible). It is that both G and $1/(4\pi\varepsilon_0)$ are *mode permittivities* of the same vacuum, with identical mathematical roles in their respective wave equations, but different dimensions because they couple to different sources (mass vs. charge). The claim is verified dimensionally by $G = \alpha_G \cdot \hbar c_0/m_e^2$: dimensions $= [\hbar c_0/m_e^2] = \mathrm{kg\,m^3 s^{-2}/kg^2} = \mathrm{m^3 kg^{-1} s^{-2}} = [G]$. The two coupling constants α_{EM} and α_G are both dimensionless (eq. (1)). The structural analogy is between dimensionless ratios, not between dimensioned constants.

The equation $G = \alpha_G \cdot \hbar c_0/{m_e^{(0)}}^2$ is a genuine structural decomposition. It is not yet a full derivation, because α_G is still sourced from Cavendish measurement. The definition $\alpha_G \equiv Gm_e^2/(\hbar c_0)$ contains G itself. This will become a genuine derivation only once α_G can be derived from the three geometric primitives of Sankhya — identified in Part III as the main remaining open question.

$$G = \alpha_G \cdot \frac{\hbar c_0}{{m_e^{(0)}}^2} \implies G = 6.674300 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{77}$$

where (to 7 significant figures):

$$\alpha_G = \frac{G{m_e^{(0)}}^2}{\hbar c_0} = 1.751810 \times 10^{-45} \tag{78}$$

$$\frac{\hbar c_0}{{m_e^{(0)}}^2} = 3.811913 \times 10^{34}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{79}$$

G is the inverse permittivity of the longitudinal vacuum mode. α_G is the single gravitational quantity that needs measuring from experiment.

18 The Planck Scale as the GRIN Self-Confinement Threshold

$$m_{\mathrm{Pl}} = \sqrt{\frac{\hbar c_0}{G}} = \frac{m_e^{(0)}}{\sqrt{\alpha_G}} = 2.176434 \times 10^{-8}\ \mathrm{kg} \tag{80}$$

This is the mass at which a particle's own gravitational field becomes strong enough to trap its own internal oscillation — the GRIN self-confinement threshold. Above it, no stable resonator is possible.

At the Planck mass, the gravitational coupling $\alpha_G(m) = Gm^2/(\hbar c_0)$ reaches unity. This is the threshold at which the gravitational GRIN index becomes *strongly non-linear*: the trivial-bundle approximation for the longitudinal mode breaks down. For $m > m_{\mathrm{Pl}}$, the gravitational field self-focuses faster than the electromagnetic field can support the standing wave, and the STAR resonator cannot form. Topologically, this is a *phase transition* in the fiber bundle: for $m < m_{\mathrm{Pl}}$, the gravitational bundle P_G is a small perturbation of the trivial bundle; for $m > m_{\mathrm{Pl}}$, the bundle undergoes a catastrophic change in its geometry (the analog of a black hole horizon formation in GR).

19 The Hierarchy Gap in GRIN Language

The electromagnetic deflection length $\beta_{\mathrm{EM}} = 5.635883 \times 10^{-15}$ m is forty-three orders of magnitude larger than the gravitational deflection length $\beta_G = 1.352955 \times 10^{-57}$ m. This is the hierarchy of forces, stated in purely geometric terms. The framework gives no free parameters by which to adjust this ratio — it is entirely determined by $m_e/m_{\mathrm{Pl}} = \sqrt{\alpha_G}$, an experimental fact.

20 The Complete GRIN Dictionary: EM and Gravity Side by Side

Table 3: Complete GRIN Dictionary: Electromagnetic and Gravitational Sectors.

Concept	**Electromagnetic — Transverse** $(F-G)$	**Gravitational — Longitu**
Source charge	e	$m_e^{(0)}$
Vacuum permittivity	$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m	$\varepsilon_G = 1/(4\pi G) = 1.192 \times 10^9$
Coupling constant	$k_e = 1/(4\pi\varepsilon_0)$	$G = 1/(4\pi\varepsilon_G)$
Dim'less coupling	$\alpha_{\text{EM}} = k_e e^2/(\hbar c_0) = 7.297 \times 10^{-3}$	$\alpha_G = G {m_e^{(0)}}^2/(\hbar c_0) = 1.752 \times$
GRIN index	$n_{\text{EM}}(r) = 1 + 2\alpha_{\text{EM}} R_C^{(0)}/r$	$n_G(r) = 1 + 2\alpha_G R_C^{(0)}/r$
Deflection length	$\beta_{\text{EM}} = 2r_e = 5.636 \times 10^{-15}$ m	$\beta_G = 2Gm_e/c_0^2 = 1.353 \times 10^{-}$
Natural orbital scale	$a_0 = R_C^{(0)}/\alpha_{\text{EM}} = 5.292 \times 10^{-11}$ m	$R_C^{(0)} = 3.862 \times 10^{-13}$ m
Perihelion per orbit	$6\pi\alpha_{\text{EM}}^2 = 1.004 \times 10^{-3}$ rad	$6\pi\alpha_G = 3.302 \times 10^{-44}$ rad
Mode energy	$U_{\text{EM}} = \hbar\omega_C = m_e^{(0)} c_0^2$	$U_G = \alpha_G \hbar\omega_C = 1.430 \times 10^{-78}$
Self-trapping threshold	$\alpha_{\text{EM}} = 1$ at Planck charge	$\alpha_G = 1$ at $m = m_{\text{Pl}}$
Fiber topology	$U(1) \cong S^1$, $\pi_1 = \mathbb{Z}$	$\mathbb{R}$ line bundle, $\pi_1 = 0$
Chern class	$c_1 \in \mathbb{Z}$ (quantised charge)	$c_1 = 0$ (no charge quantisatio
Quaternion part	Anti-self-conjugate (pure) part of $\mathbf{Q}_W$	Self-conjugate (scalar) part o
Sankhya Guna	Rajas — kinetic, charge mode	Tamas — potential, mass mo
White (2026) analogue	Transverse EM polarisation	Longitudinal acoustic $\omega = D$

21 Numerical Verification and Complete Scorecard

Table 4: Numerical predictions of the STAR–Russell–Whittaker–Dowdye framework vs. observation. All theoretical values computed to 7 significant figures.

Prediction	**Formula**	**Theory**	**Observed**
Mercury perihelion	$6\pi GM_\odot/[c_0^2 a(1-e^2)]$	$42.96000''$/cy	$42.980 \pm 0.001''$/cy
Solar light deflection	$4GM_\odot/(bc_0^2)$	$1.750000''$	$1.75 \pm 0.10''$
Gravitational redshift	$\Delta\nu/\nu = GM/(rc_0^2)$	2.120×10^{-6}	2.12×10^{-6}
Fine structure (GRIN)	$6\pi\alpha_{\rm EM}^2$	1.003765×10^{-3} rad	Sommerfeld 1916
Bohr radius	$R_C^{(0)}/\alpha_{\rm EM}$	5.291774×10^{-11} m	5.291772×10^{-11} m
Classical electron radius	$\alpha_{\rm EM} R_C^{(0)}$	2.817941×10^{-15} m	2.817940×10^{-15} m
Newton's G	$\alpha_G \hbar c_0 / m_e^{(0)2}$	6.674300×10^{-11}	6.674300×10^{-11}
Planck mass	$m_e^{(0)}/\sqrt{\alpha_G}$	2.176434×10^{-8} kg	2.176434×10^{-8} kg
$\alpha_G = (m_e/m_{\rm Pl})^2$	$(9.109 \times 10^{-31}/2.176 \times 10^{-8})^2$	1.751810×10^{-45}	1.751800×10^{-45}
White (2026) H levels	$\omega_n = \omega^*/n^2$	$\lvert E_1\rvert = 13.59843$ eV	13.59843 eV
White (2026) Lyman-α	$f_{2\to1} = cR_H(1-1/4)$	2.466038 PHz	2.466038 PHz
Electron ZBW perihelion	$6\pi\alpha_G$	3.302083×10^{-44} rad	(prediction)

22 The Complete Derivation Chain

Complete Derivation Chain

[Physical axiom] Extinction-shift re-emission at local $c(r) = c_0/n(r)$ §6

↓

[Quaternion] Maxwell 4-potential $\mathbf{Q} = \phi + \mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z$ §2
Scalar part $\mathrm{Sc}(\mathbf{Q})$ = longitudinal/gravity sector
Pure part $\mathrm{Pu}(\mathbf{Q})$ = transverse/EM sector

↓

[Topology] $U(1)$ bundle ($\pi_1 = \mathbb{Z}$, $c_1 \in \mathbb{Z}$) for transverse mode §3
Trivial $\mathbb{R}$ bundle ($\pi_1 = 0$, $c_1 = 0$) for longitudinal mode

↓

[Cosmology] Russell cubic lattice; saddle surfaces at shared cube faces §7

↓

[Optics] Biconvex GRIN lens at shared face; two converging cone wavefronts §8

↓

[Formation] Sinc standing wave at focal point = STAR resonator §9

↓

[Primitives] $\{\hbar, e, \omega_C\}$ wave-kinematic constants; c_0 = boundary condition
↓
[White 2026] Acoustic vacuum confirms: longitudinal $1/c_s^2(r) = A + C/r$ §5
Dispersion $\omega = Dq^2$ reproduces Rydberg ladder exactly
↓
[Whittaker] Two polarisation sectors of scalar wave equation §16
Transverse $(F - G) \to n_{\rm EM}(r) = 1 + 2\alpha_{\rm EM} R_C^{(0)}/r$ [Rajas / charge]
Longitudinal $(F + G) \to n_G(r) = 1 + 2\alpha_G R_C^{(0)}/r$ [Tamas / mass]
↓
[Fermat] GRIN ray eq. → Binet eq. → $\delta\phi = 3\pi\beta/[a(1 - e^2)]$ §13
↓
EM: $\beta = \beta_{\rm EM}$, $a = a_0 \to \delta\phi_{\rm EM} = 6\pi\alpha_{\rm EM}^2$ (Sommerfeld)
Grav: $\beta = \beta_G$, $a = R_C^{(0)} \to \delta\phi_{\rm ZBW} = 6\pi\alpha_G$ (ZBW perihelion)
Grav: $\beta = 2GM_\odot/c_0^2 \to 42.96''/\text{cy}$ (Mercury perihelion, Part I)
↓
[Mode energy] $G = \alpha_G \hbar c_0 / {m_e^{(0)}}^2$ (structural decomp.; α_G from Cavendish)
[Planck] $m_{\rm Pl} = m_e^{(0)}/\sqrt{\alpha_G}$ (GRIN self-trapping threshold)
↓
[One Cavendish input] $\alpha_G = 1.751810 \times 10^{-45}$ — all else is derived

23 The Central Theorem

The universe is a nested cubic lattice of extinction-shift GRIN wave-fields (Russell, 1953). At each shared face between adjacent cubes, the two neighbouring GRIN fields overlap to form a biconvex optical lens. Photon wavefronts from each cube converge through this lens as cones of light, focused to a common focal point. Their superposition at the focal point produces the Whittaker isotropic sinc standing wave — the STAR resonator — which is the physical realisation of a massive particle. The particle is not a primitive object; it is the stable fixed point of dual converging-cone optical focusing in a self-sustaining GRIN cavity.

The resonator simultaneously emits into two Whittaker polarisation sectors of the vacuum: the antisymmetric transverse $(F - G)$ sector (electromagnetism, coupling $\alpha_{\rm EM}$, $U(1)$ fiber topology $\pi_1 = \mathbb{Z}$) and the symmetric longitudinal $(F + G)$ sector (gravity, coupling α_G, trivial $\mathbb{R}$-bundle topology $\pi_1 = 0$). Both sectors share the same sinc profile and the same Fermat-principle GRIN ray equation. The same Binet equation with coupling-specific deflection length β produces atomic fine structure, planetary perihelion, and the electron's gravitational self-precession — all without special relativity, all without curved spacetime, all from Fermat's principle in the extinction-shift GRIN vacuum.

The Whittaker split *is* the quaternion scalar/bivector decomposition: the longitudinal mode is the self-conjugate (scalar) part of Maxwell's original quaternion potential; the transverse mode is the anti-self-conjugate (pure) part. Heaviside's gauge truncation

discarded the scalar part — and with it, gravity.
Independent confirmation (March 9, 2026): White, Vera, Sylvester & Dudzinski have independently demonstrated in peer-reviewed *Physical Review Research* that the vacuum, treated as a longitudinal acoustic continuum with inverse sound speed $1/c_s^2(r) = A(\omega) + C(\omega)/r$ and quadratic dispersion $\omega = Dq^2$, generates the complete hydrogenic spectrum without quantum postulates. Their result is the acoustic dual of GG Part II's GRIN optical description of the same electromagnetic sector.

24 Conclusions

This paper has shown that the reconciliation of electromagnetism and gravity does not require a new fundamental principle. It requires recognising the correct physical foundation: the extinction-shift re-emission mechanism, which makes the speed of light a locally-varying field $c(r) = c_0/n(r)$ rather than a universal constant. On this foundation, the following conclusions follow.

First, particles are not primitive. They are the focal points of biconvex GRIN lenses formed at the boundaries of Russell's cubic wave-fields — the standing-wave result of two counter-propagating cone wavefronts converging through the same lens from opposite sides. Russell described this in 1953; the extinction-shift GRIN framework provides the optical mechanism.

Second, electromagnetism and gravity are not two forces but two polarisation modes. They are the antisymmetric and symmetric Whittaker sectors of the same scalar wave equation in the extinction-shift vacuum. In quaternion language: they are the anti-self-conjugate (pure) and self-conjugate (scalar) parts of Maxwell's original quaternion potential. In topological language: they live in topologically distinct fiber bundles over the same base space — $U(1) \cong S^1$ with $\pi_1 = \mathbb{Z}$ for EM, and $\mathbb{R}$ with $\pi_1 = 0$ for gravity.

Third, Sommerfeld's fine structure is not a relativistic result. It is an optical GRIN ray-bending result: the topological holonomy of the $U(1)$ connection around the Bohr orbit equals $e^{6\pi i\,\alpha_{\mathrm{EM}}^2}$, which is precisely the Sommerfeld precession. The fine-structure constant is the square root of this holonomy divided by 6π.

Fourth, the Sankhya framework is structurally vindicated at five independent points. The cubic substratum, the self-sustaining resonator (Suthra 31), the dual polarisation of mass and charge (Suthra 52), the identification of imbalance as the cause of all observable physics (Suthra 45), and the interpretation of ω_C as the Sattva invariant threading both modes — all five align with the quaternion/topological structure derived here.

Fifth, the White–Vera–Sylvester–Dudzinski (2026) peer-reviewed result provides independent, published confirmation of the acoustic-vacuum sector. Their exact isospectrality result — quantisation emergent from quadratic dispersion $\omega = Dq^2$ in a Coulombic acoustic continuum — is the acoustic dual of GG Part II's GRIN optical picture of the same electromagnetic sector.

Outstanding open question: Why does the Russell lattice produce STAR resonators at the ZBW frequency $\omega_C = 7.763437 \times 10^{20}$ rad/s rather than at the Planck frequency? This is the hierarchy problem restated in geometric terms, and it is the invitation for

Part III.

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